Deterministic vs. stochastic models

- In **deterministic** models, the output of the model is fully determined by the parameter values and the initial conditions.

- **Stochastic** models possess some inherent randomness. The same set of parameter values and initial conditions will lead to an ensemble of different outputs.

- Obviously, the natural world is buffeted by stochasticity. But, stochastic models are considerably more complicated. When do deterministic models provide a useful approximation to truly stochastic processes?

Demographic vs. environmental stochasticity

- **Demographic stochasticity** describes the randomness that results from the inherently discrete nature of individuals. It has the largest impact on small populations.

- **Environmental stochasticity** describes the randomness resulting from any change that impacts an entire population (such as changes in the environment). Its impact does not diminish as populations become large.

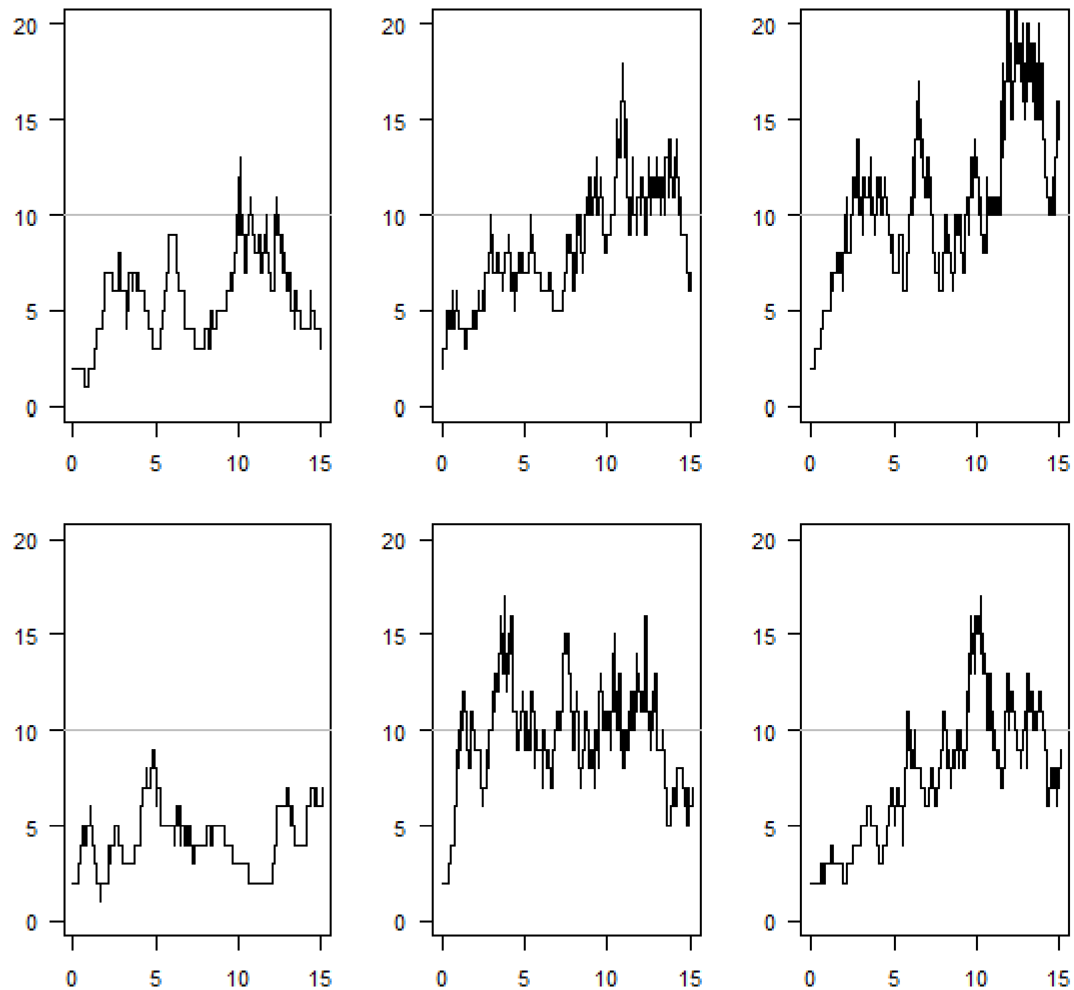
Stochastic models, brief mathematical considerations

- There are many different ways to add stochasticity to the same deterministic skeleton.

- Stochastic models in continuous time are hard.

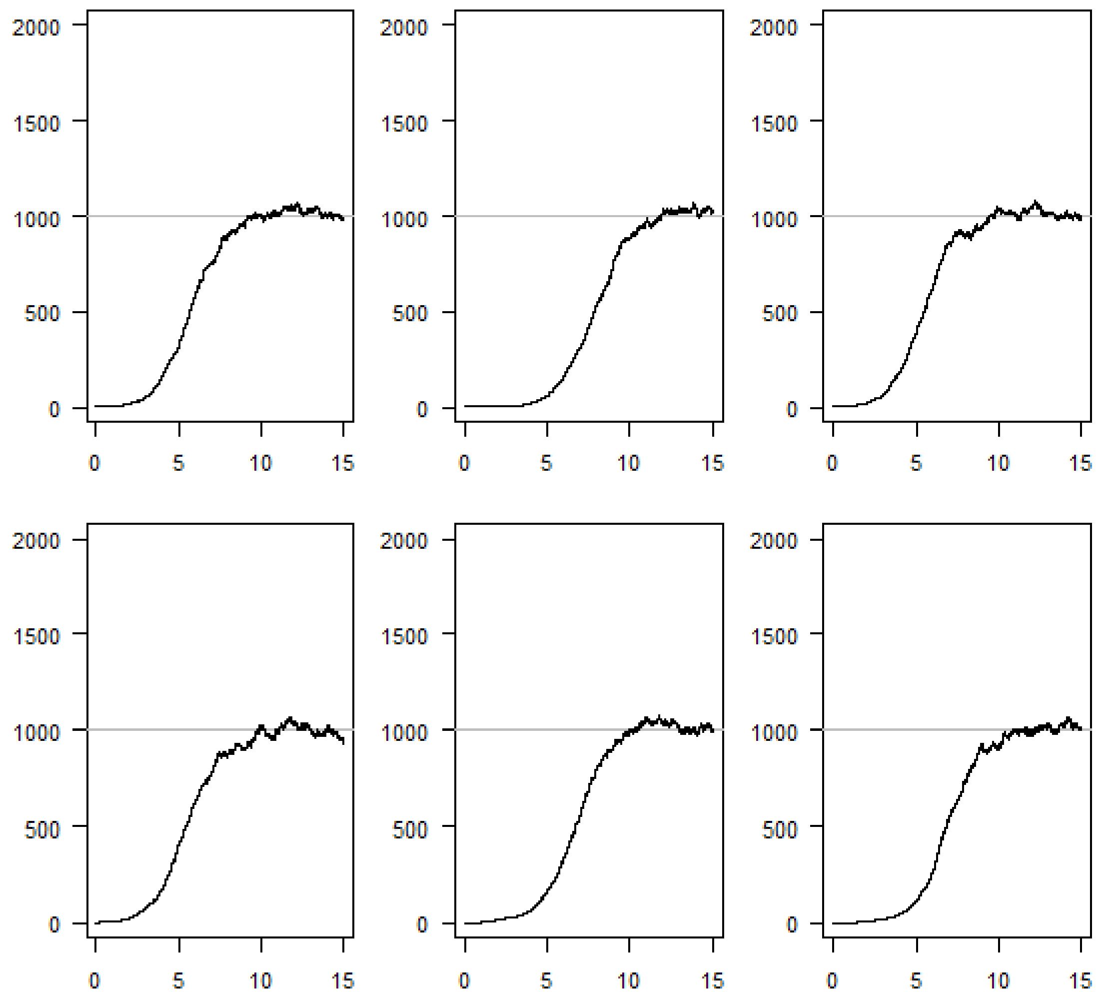
- Gotelli provides a few results that are specific to one way of adding stochasticity.

Demographic stochasticity has its biggest impact on small populations



6 runs of stochastic logistic growth model, carrying capacity = 10

Demographic stochasticity has its biggest impact on small populations



6 runs of stochastic logistic growth model, carrying capacity = 1000

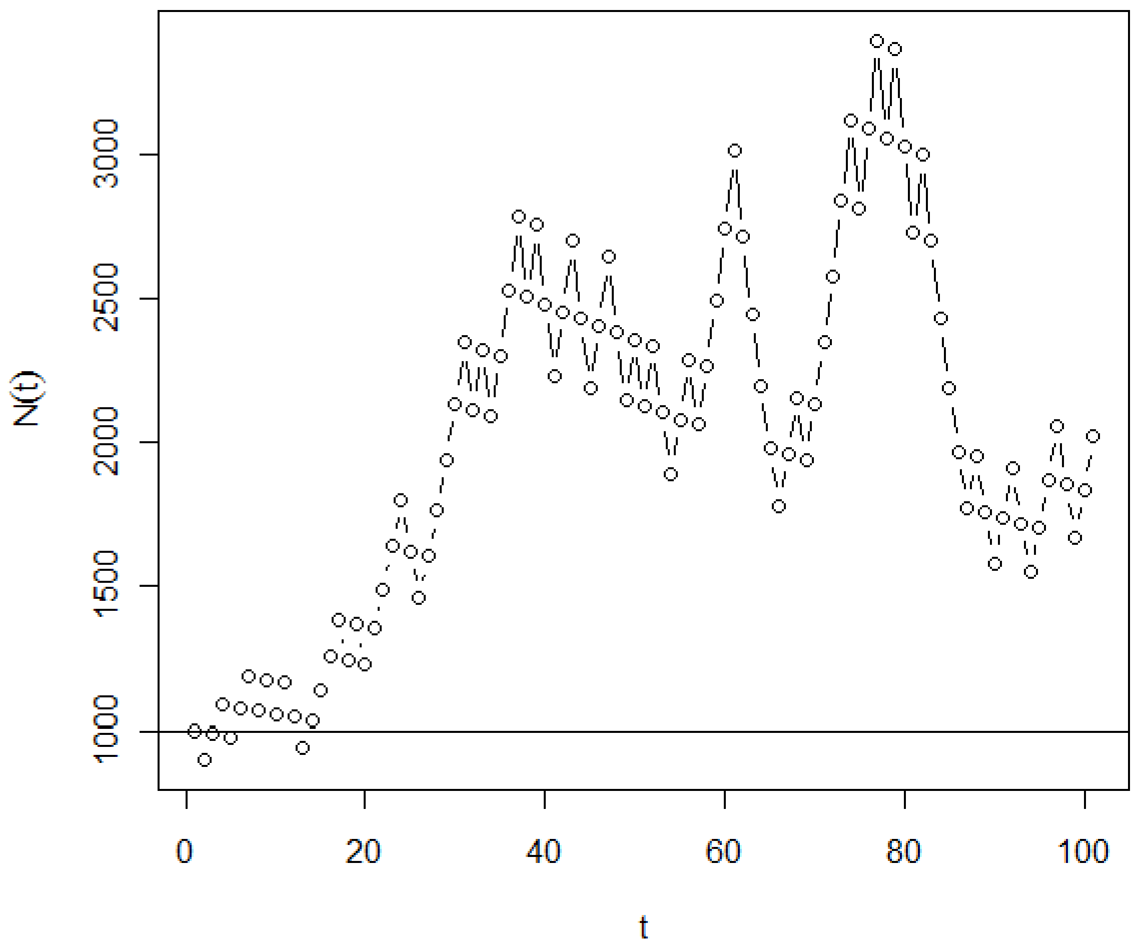
A stochastic version of the geometric population growth model

$$N_{t+1} = \lambda(t) N_t$$

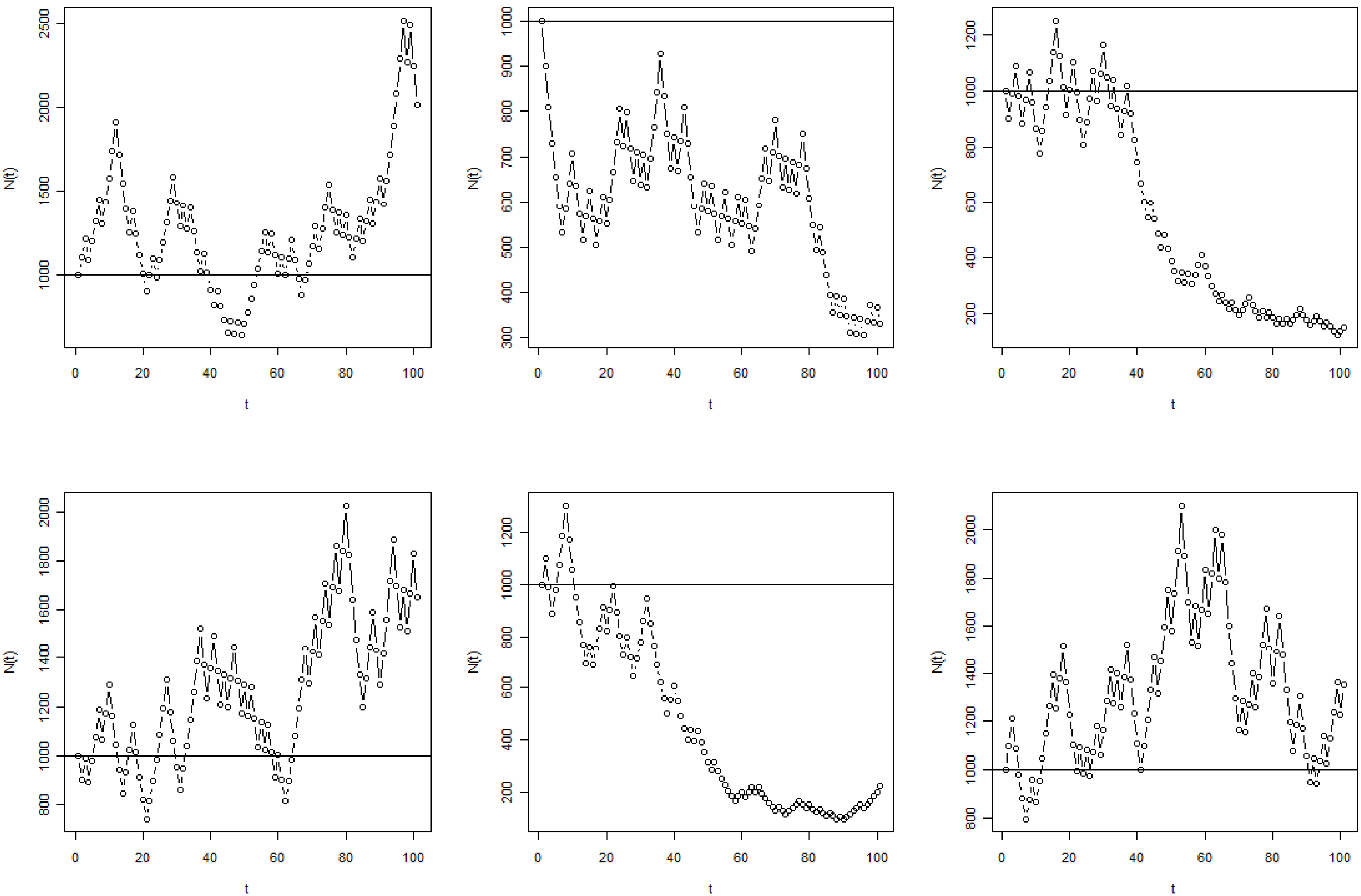
- Suppose that λ has the following probability distribution:

  = 0.9 with probability ½
  = 1.1 with probability ½

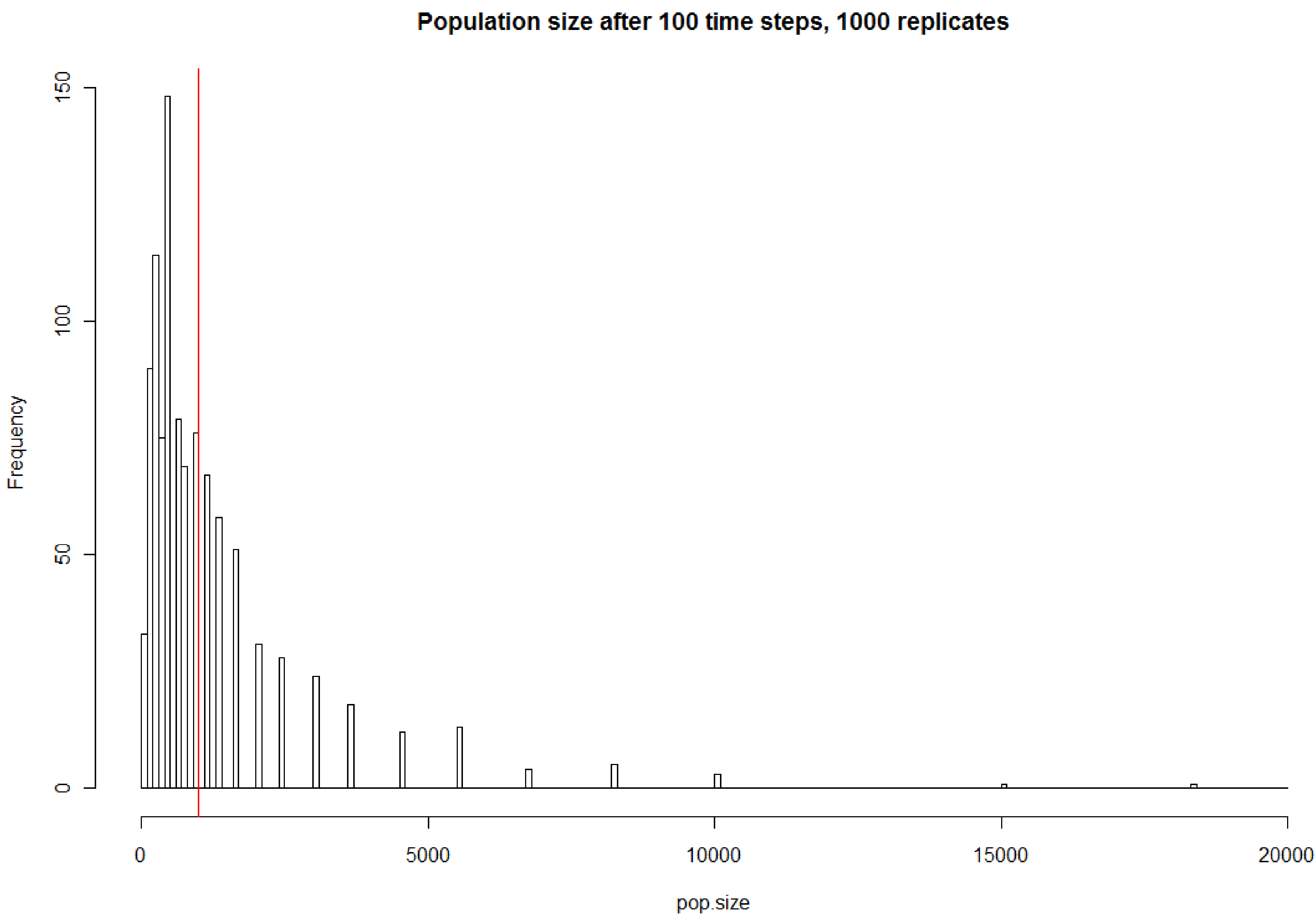
What are typical behaviors of this population?

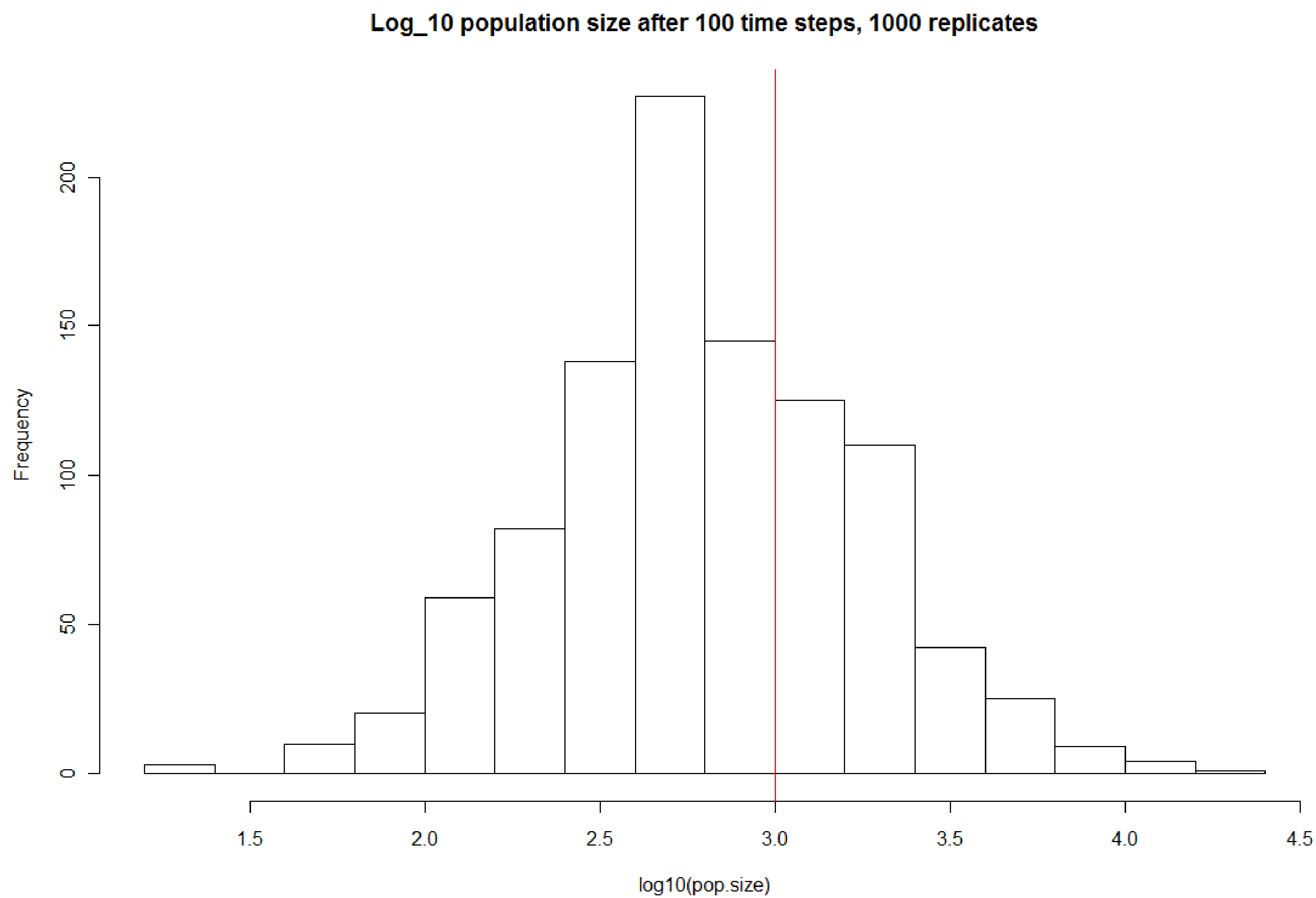


N(t)
1000
1500
2000
2500
3000
0
20
40
60
80
100
t



N(t)
t
1000
1500
2000
2500
300
400
500
700
800
900
200
400
800
1200
600
1400
1600
1800
0
20
40
60
80
100


Population size after 100 time steps, 1000 replicates
Frequency
0
50
100
150
0
5000
10000
15000
20000
pop.size


Log_10 population size after 100 time steps, 1000 replicates
Frequency
0
50
100
150
200
1.5
2.0
2.5
3.0
3.5
4.0
4.5
log10(pop.size)

- Stochastic population growth yields log-normally distributed population sizes

- Many small populations, few large ones

- The rate of change of the average population size overestimates the “typical” growth rate experienced by most populations.

- Mathematical details, for those interested:

$$E\left[\left(\frac{N_t}{N_0}\right)^{\frac{1}{t}}\right] < \left(\frac{E\left[N_t\right]}{N_0}\right)^{\frac{1}{t}} = E\left[\lambda\right]$$

Average rate of change of the population

Rate of change of the average population

- (this is Jensen's inequality)