Magic Number :- A magic number is a number of nucleons (either protons or neutrons, seperately) such that they arranged into complete shells within the atomic nucleus. The seven most widely recognized magic numbers are

2, 8, 20, 28, 50, 82 and 126.

Nuclei which have neutron number and proton numbers each equal to one of the magic numbers are called **doubly magic**.

Example :- $_2He^4$, $_8O^{16}$, $_{20}Ca^{40}$, $_{82}Pb^{208}$
(Z - 2, N - 2), (Z - 8, N - 8) (Z - 20, N - 20) (Z - 82, N - 126)

There are some special features of magic nuclei —

⇒ i) The neutron or proton seperation energy peaks if N or Z is equal to a magic number.

⇒ ii) There are more stable isotopes if Z is a magic no. and more stable isotones if N is a magic no.

⇒ iii) The energies of the excited states are much higher than the ground state if either N or Z or both are magic No.

⇒ iv) Elements with Z equal to a magic no. have a larger abundance and hence more stable than those of near by elements.

SHELL MODEL

The basic assumption of the liquid drop is that each nucleon in a nucleus interacts only with its nearest neighbours, like a molecule in a liquid. At the other end, we also know that each nucleon interacts chiefly with a general force field produced by all the other nucleons. This latter situation can be compared with that of electrons in an atom, where only certain quantum states are permitted and no more than two electrons being fermions can occupy each state.

⇒ In an atom, electrons ~~may be thought of~~ can occupy positions in 'shells' designated by the various principal quantum numbers. It is ~~a~~ seen that atoms with 2, 10, 18, 36, 54, and 86 electrons have all their electron shell completely filled. Such electrons structures have high binding energies and are exceptionally stable.

⇒ The same kind of effect is observed with nuclei. Nuclei that have 2, 8, 20, 28, 50, 82, and 126 neutrons or ~~and~~ protons are more abundant than other nuclei of similar mass no, suggesting that their structures are more stable.

⇒ Another thing is the observed pattern of nuclear electric quadrupole moments which measures how much ~~no~~ nuclear charge distribution depart from sphericity. A ~~speh~~ spherical nucleus has no quadrupole moment, while one shaped like football (Prolate shape) has positive moment and one shaped like pumpkin (oblate shape) has negetive moment. It is seen that all ~~mir~~ magic nuclei have spherical shape as their quadrupole moment is zero.

All ~~this~~ these facts lend support to the shell structure of nuclei.

The shell model of the nucleus is an attempt to account for the existance of magic numbers and certain other nuclear properties in terms of nucleon behaviour in a common force field.

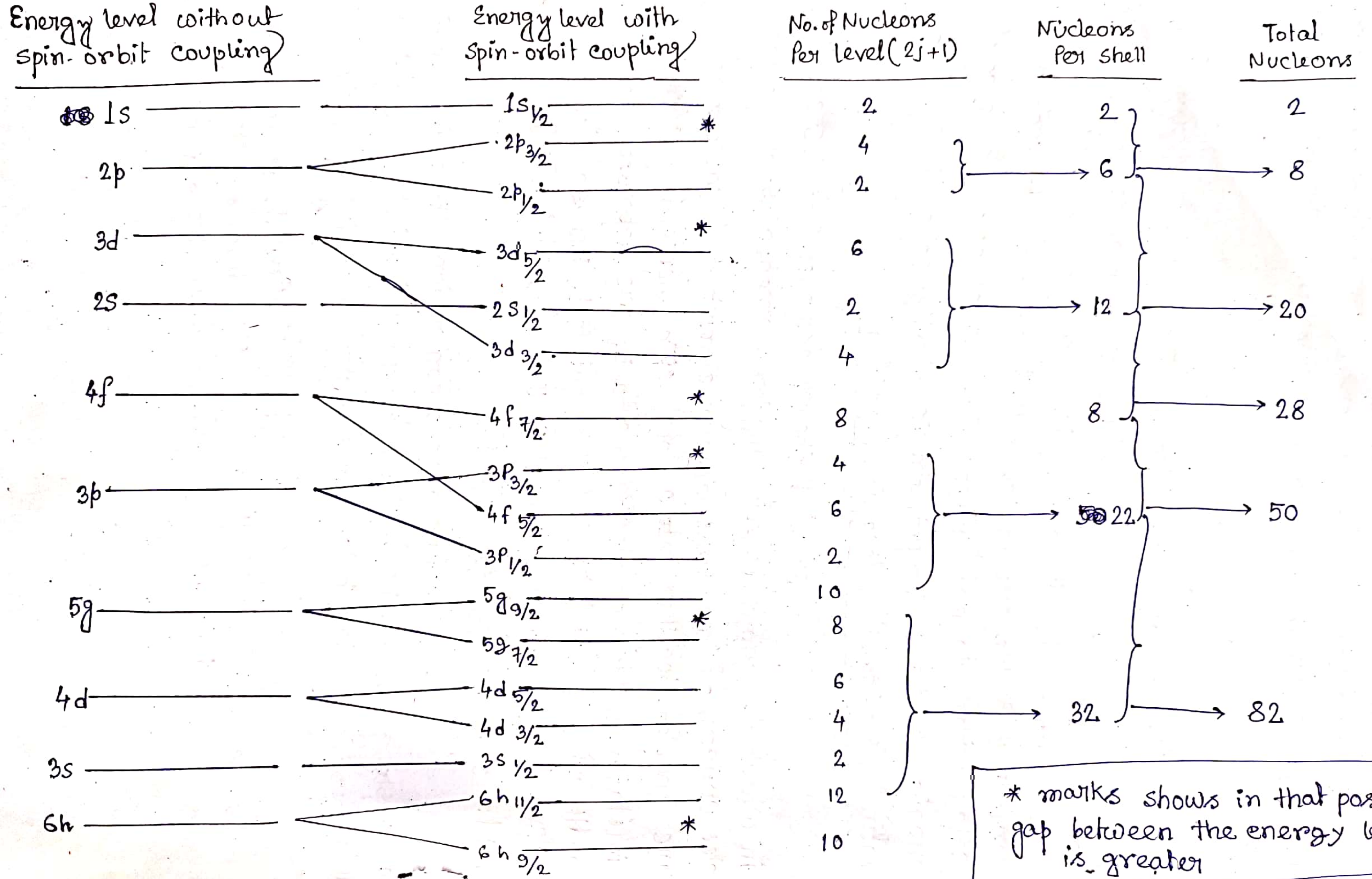
How Magic Number Arise :- The potential energy fn. for a nucleus is not known. A resonable guess on the basis of the nuclear density curves is a square well with rounded corner. But the energy levels that come from such a calculation do not agree with the observed sequence of magic numbers. The problem is finally solved by incorporating spin-orbit interaction (Coupling).

The shell theory assumes that LS coupling in which the intrinsic spin angular momenta S_i of the particles concerned (neutrons form one group and protons another) are coupled together into a total spin momentum S. Similarly orbital angular momenta L_i are seperately coupled togather into a total orbital momentum L then S and L coupled to form total angular momentum J of magnitude $\sqrt{j(j+1)}\,\hbar$, holds only for lighter nuclei.

After a transition region, where intermediate coupling scheme holds, the heavier nuclei exhibit jj coupling. This coupling scheme holds for the great majority of nuclei.

When an appropiate strength is assumed for the spin-orbit interaction, the energy level of either class of nucleon fall into the sequence shown in the figure

⟶



* marks shows in that positions gap between the energy level is greater

The spin-orbit interaction splits each state of given j into $2j+1$ substates, since there are $2j+1$ allowed orientations of J_i. Large energy gap appear in the spacing of the levels at interval that are consistant with the notion of seperate shells. The number of available nuclear states in each shell is respectively 2, 6, 12, 8, 22, 32, ...

Hence shells are filled when there are 2, 8, 20, 28, 50, 82, 126 Neutrons or protons in a nucleous.

That's how magic number arises where the nuclei is much more stable.

☐ Now if we look into the fig. then we can see that all levels split except 's' level. Since all level split according to j value. For 's' level $l=0$ and $s=\pm 1/2$. therefore $j=0+\frac{1}{2}=\frac{1}{2}$ and $j=0-\frac{1}{2}$ (Not allowed)

But for other levels say 2P

$$l=1$$
$$s=\pm\frac{1}{2}$$
$$j=1+\frac{1}{2}=\frac{3}{2}$$
$$=1-\frac{1}{2}=\frac{1}{2}$$

So, every level can split except 's' level.

☐ We know the spectroscopic Notation

$$n\,l_j^{2j+1}$$

Where $n \rightarrow$ total quantum no.

$j \rightarrow$ total angular momentum quantum no.

$l \rightarrow$ Orbital " " "

(s, p, d, f, g, ... correspondingly $l = 0, 1, 2, 3, 4, ...$)

$2j+1 \rightarrow$ No. of nucleon, level can take (Sometimes it is called multiplicity of state)

☐ Energy levels in the nuclei is filled in the following sequence

$$1s^2_{1/2}\ 2p^4_{3/2}\ 2p^2_{1/2}\ 3d^6_{5/2}\ 2s^2_{1/2}\ 3d^4_{3/2}\ 4f^8_{7/2}\ 3p^4_{3/2}\ 4f^6_{5/2}\ 3p^2_{1/2}\ 5g^{10}_{9/2}\ 5g^8_{7/2}\ 4d^6_{5/2}\ 4d^4_{3/2}\ 3s^2_{1/2}\ 6h^{12}_{11/2} —$$

(As in the fig. I suggest you to remember this sequence upto 50 shown by dotted bracket)

☐ The shell model accounts for several nuclear phenomena in addition to magic number. we can determine Spin and parity of a nuclei.

☐ A nuclei with even no. of proton and neutron (i.e even- even nuclei) has <u>Zero</u> spin and <u>+ve</u> parity

☐ For a odd-even or even-odd nuclei spin and parity is determined as follows,

Let us take an <u>example</u>

$_{13}Al^{27}$ proton no. (Z) = 13

Neutron no. (N) = 14

Here, proton no. is odd Hence it has a unpaired proton But neutron no. is even hence it has no unpaired neutron

$$\therefore 13P \rightarrow 1s^2_{1/2}\ 2p^4_{3/2}\ 2p^2_{1/2}\ 3d^5_{5/2}$$

Hence its 3d level is unfilled with one ~~up~~ unpaired proton Hence its spin is $\frac{5}{2}$

and parity is determined by $(-1)^l$

Here $l = 2$ $\therefore (-1)^2 = 1$ (+ve)

$\therefore$ spin and parity of $_{13}Al^{27} = \frac{5}{2}^+$

Basically we have to see which one is odd (Proton no. or Neutron no.) then we have to see which energy level of the corresponding one is unfilled then j-value of that level gives spin and finding l we will have parity.

Limitation :- ~~Insipe~~ Inspite of many advantages shell model has some limitation too.

i) The four stable nuclei $_1H^2$, $_3Li^6$, $_5B^{10}$, $_7N^{14}$ do not fit into this model.

ii) It also fails to explain the observed large quadrupole moment of odd-A nuclei.

Home work :- Find the spin and parity of the following nuclei.

i) $_{21}Sc^{45}$ ii) $_{29}Cu^{63}$ iii) $_{32}Ge^{63}$ iv) $_{30}Zn^{67}$

(Spin = Ground state angular momenta)

Radioactivity

Despite the strength of the forces that hold nucleons together to form an atomic nucleus, many nucleis are unstable and spontaneously change into other nucleis by radio active decay. And all nuclei can be transformed by reactions with nucleons or other nuclei that collide with them.

Radio active decay :- Radioactivity plays a significant role in the development of nuclear physics. Three ~~feae~~ features of radioactivity are extraordinary from the perspective of classical physics. –

⇒ 1. When a nucleus undergoes alpha or beta decay, its atomic no (Z) changes and it becomes the nucleus of the different element. Which means the elements are not immutable

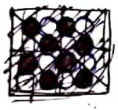
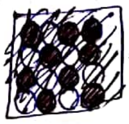
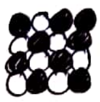
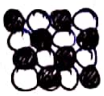
⇒ 2. The energy liberated during radioactive decay comes from within individual nuclei without any external excitation How can this happen ?? this puzzle was solved by Einstein (Equivalence of mass and energy).

⇒ 3. Radioactive decay is a statistical process that obeys the law of chace. No cause-effect relationship is involved in the decay of a particular nucleus, only a certain probability per unit time.

Classical physics can not account for such behaviour although it fits naturally into the framework of quantum physics.

Different kind of Radioactive decay:- The early experimenters, among them Rutherford and his co-workers distinguished three components in the radiation from radionuclides. These components are alpha ($_2He^4$), beta (e^-), and gamma (high energy photon). Later, positron emission (e^+) and electron capture are added to the list of decay modes.

Therefore, the five ways in which an unstable can decay with the reason for their instability is shown by diagram.

	Original Nucleus	Decay event	Final Nucleus	Reason for Instability
Gamma decay		Emission of gamma ray reduces energy of nucleus		Nucleus has excess Energy
Alpha decay		 Emission of alpha particle reduces Size of nucleus		Nucleus is too large.
Beta decay	 	o = • + ● (e⁻) Emission of electron by neutron in nucleus changes the neutron to a proton.	 	Nucleus has too many neutrons relative to number of protons.
Electron capture		(e⁻) • + ● = o Capture of electron by proton in nucleus changes it into neutron		Nucleus has too many protons than neutrons.
Positron emission		● = ∘ + o (e⁺) Emission of positron by proton in nucleus changes it into neutron		Nucleus has too many protons than neutrons.

● → Proton

o → Neutron

• → electron (e^-)

∘ → Positron (e^+)

Homework

The helium isotope $_2He^6$ is unstable. What kind of decay would you expect it to undergo ??

Radioactive decay law :-

The rate of radioactive disintegration that is the number of atoms (nuclides) that break up at any instant of time t is directly proportional to the number N_t of active nuclides present in the sample under study at that instant.

Decay Equation :- From the above law,

$$-\frac{dN_t}{dt} \propto N_t$$

$$\Rightarrow \frac{dN_t}{dt} = -\lambda N_t \quad [\lambda = \text{Constant of proportionality}]$$

negetive sign indicates that N_t decreases with t.

$$\therefore \frac{dN_t}{N_t} = -\lambda dt$$

$$\Rightarrow \ln N_t = -\lambda t + \ln A$$

if initial no. of nuclides is N_0 then

$$\boxed{N_t = N_0 e^{-\lambda t}}$$

This shows that no. of active nuclides decreases exponentially with time, that means an infinite is needed for the radioactivity to completely disappear.

Deduction from probability :- We have already seen that radioactivity is a statistical process: Therefore the decay equation can also be deduced using probability concept.

The probability p for an atom (Nuclide) to disintegrate in an time interval Δt depends only on the length of time interval and for sufficiently short intervals is proportional to Δt.

$$p \propto \Delta t$$

$$\therefore \; p = \lambda . \Delta t \qquad \lambda = \text{disintegration constant.}$$

Then the probability that the nuclide will not disintegrate in this interval Δt is $p' = 1 - p = 1 - \lambda \Delta t$

The probability that the atom will not disintegrate in second interval Δt is also same.

Therefore the probability that this atom will survives both intervals is $(1-\lambda\Delta t)^2$. For, n such intervals probability of survival is $(1-\lambda\Delta t)^n$

$$\therefore \; (1-\lambda\Delta t)^n = \left[1 - \lambda\left(\frac{t}{n}\right)\right]^n \qquad [t = \Delta t + \Delta t + \cdots = n.\Delta t]$$

$\therefore$ The probability that the atom will remain unchanged after a time t is

$$\lim_{n \to \infty} \left(1 - \lambda \frac{t}{n}\right)^n = e^{-\lambda t}$$

We may interpret this statistically, if N_0 be the initial no. of atoms and N_t is the number of <u>unchanged</u> atoms after t then

$$\frac{N_t}{N_0} = e^{-\lambda t}$$

$$\therefore \; N_t = N_0 e^{-\lambda t}.$$

λ depends only on the energy that is available for the transformation and parent nucleus.

Half-life :– The half life of a radioactive nuclide is defined as the time $T_{1/2}$ in which the original amount of radio active atoms is reduced by way of disintegration to half its value.

Substituting $N_t = \frac{N_0}{2}$ in decay equation

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$\therefore T_{1/2} = \ln 2/\lambda = \frac{0.693}{\lambda}$$

$$\therefore \lambda T_{1/2} = \text{Const}$$

So, $T_{1/2}$ is independent of the instant from which it is measured.

~~If a graph is~~

- **Significance** :- It describes how quickly unstable atoms undergo, or how long stable atom survive, radio active decay.

- **Average life/Mean-life** :– Average or mean life $\bar{T}$ of a radio-active element is the average lifetime of all the atoms in the given sample and is defined as the ratio of the total lifetime of all atoms to the total number of atoms.

$$\therefore \bar{T} = \frac{\int_{t=0}^{\infty} t\, dN}{\int_{N_0}^{0} dN} = -\frac{1}{N_0} \int_0^{\infty} t\, d(N_0 e^{-\lambda t})$$

$$= + \int_0^{\infty} \lambda t e^{-\lambda t} dt$$

$$= \frac{1}{\lambda}$$

$$\therefore \boxed{\bar{T} = \frac{1}{\lambda}}$$

- **Note** :- The no. of atoms is reduced by a factor $\frac{1}{2}$ after a time $T_{1/2}$. In general the number would be reduced by $(\frac{1}{2})^n$ after a time $nT_{1/2}$, that is after n half-lives.

Activity :- Activity of a radioactive sample at any instant t is defined as the number of disintegrations occurring in the sample in unit time at t, that is

$$\text{Activity } A_t = \left|\frac{dN_t}{dt}\right| = \lambda N_t = \lambda N_0 e^{-\lambda t}$$

$$\therefore A_t = A_0 e^{-\lambda t} \quad [A_0 = \text{initial activity}]$$

Activity per unit mass of a sample is called specific activity.

Home Work :-

i) How long does it take for 60% of sample of radon (Rn^{222}) to decay?

ii) Find the activity of 1.00 mg of Radon (Rn^{222}) whose atomic mass is 222 u.

iii) A piece of wood from the ruins of an ancient dwelling was found to have a C^{14} activity of 13 disintegration per minute per gram of its carbon content. The C^{14} activity of a living wood is 16 disintegration per minute per gram. How long ago did the tree die from which the wood sample came.??

iv) if M_e, M_p, and M_H are the rest masses of electron, proton and hydrogen atom in the ground state then what will be the value of M_H?

v) The activity of a radio active sample is decreased to 75% of the initial value after 30 days. What will be its half life??

vi) A particular radio isotope has a half life of 5 days. In 15 days what will be the probability of decay in percentage??