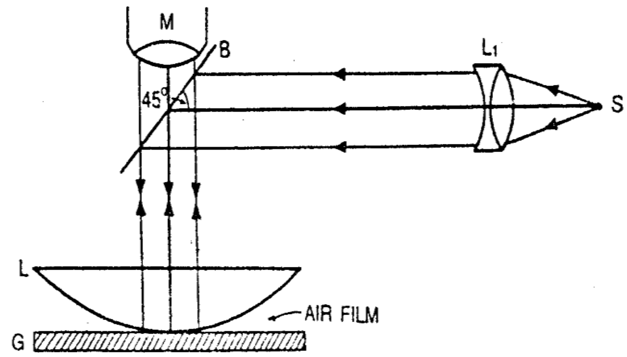
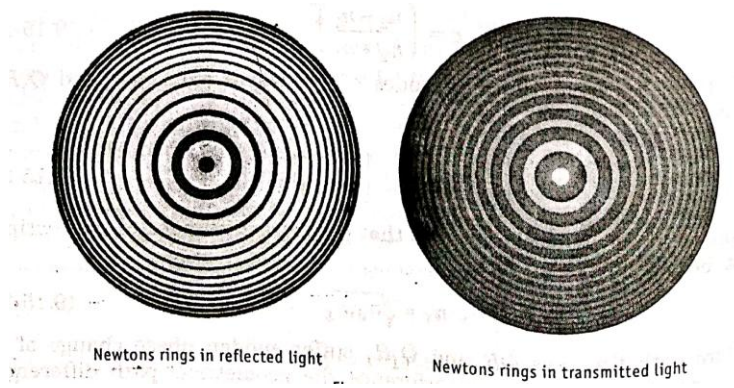
Interference of Light by division of amplitude

- **Newton's Rings:**

Newton's rings are a particular example of interference fringes formed by thin films. When a Plano-convex lens of long focal length is placed on a plane glass plate, a thin film of air is enclosed between the lower surface of the lens and the upper surface of the plate. The thickness of the air film is very small at the point of contact and gradually increases from the centre outwards. The fringes produced with monochromatic light are circular. The fringes are concentric circles, uniform in thickness. When viewed with white light, the fringes are colored. With monochromatic light, bright and dark circular fringes are produced in the air film.





(1) (2)

Fig: (1)An arrangement for observing Newton's rings. Light from an extended source S is allowed to fall on a thin air film formed between the Plano-convex lens L and the plane glass plate G. M represents a traveling microscope and B is glass plate. (2) Newton's ring in reflected and transmitted light.

- **Theory:**

A. Newton's rings with reflected light:

Newton's rings are formed due to interference when light waves reflected from convex lower surface of the lens and flat upper surface of the glass plate. Usually the Plano-convex lens has large radius of curvature and thickness of air film very small. Under such condition the optical path difference between two successive reflected waves QS_1R_1 and NS_2R_2 will be (fig-3)

$$2\mu d \pm \lambda/2 \quad \ldots\ldots\ldots\ (1)$$

d= thickness of the air film at N and $\lambda/2$ is the additional path difference due to reflection at G.

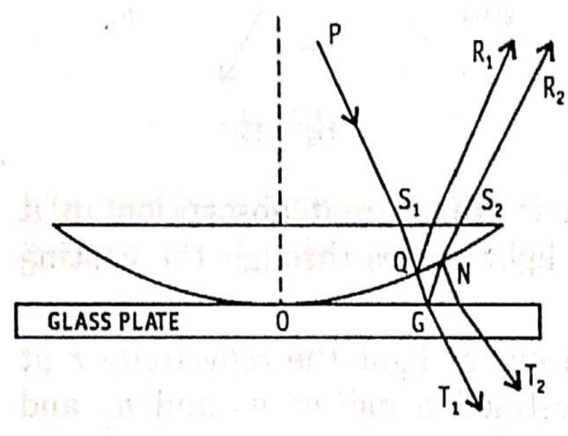


Fig.-(3)

Now, condition for constructive interference:

$$2\mu d \pm \lambda/2 = \text{even multiple of } \lambda/2$$

$$\text{Or, } 2\mu d = \text{odd multiple of } \lambda/2$$

$$= (2m+1)\lambda/2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ (2)$$

$$\text{Where } m = 0, 1, 2, 3 \ldots\ldots$$

Condition for destructive interference is:

$$2\mu d \pm \lambda/2 = \text{odd multiple of } \lambda/2$$

$$\text{Or, } 2\mu d = \text{even multiple of } \lambda/2$$

$$= 2m.\lambda/2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3)}$$

$$\text{Where } m = 0, 1, 2, 3 \ldots\ldots.$$

A fringe of a given order (m) will be along the loci of points of equal film thickness (d) and hence the fringe will be circular.

From fig-4, QQ_1 is the radius of m[th] order bright or dark ring

$$QQ_1 = r_m$$

And using Geometry, $R^2 = {r_m}^2 + (R - d)^2$

Where R= radius of curvature of the convex surface.

Since $R \gg d$, we can write,

$${r_m}^2 \approx 2Rd \quad \ldots\ldots\ldots\ldots\ldots \text{(4)}$$

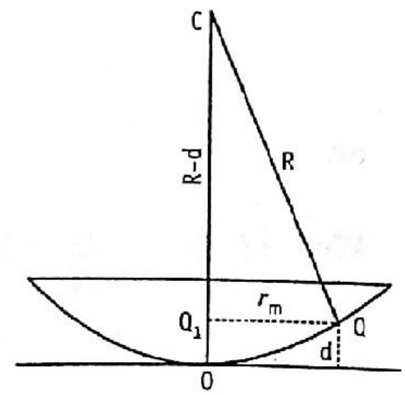


Fig-(4)

Using equation (2) for m[th] order bright fringe

$${r_m}^2 = \frac{(2m+1)\lambda R}{2\mu} \quad \ldots\ldots\ldots\ldots\ldots.(5)$$

Similarly, Using equation (3) for m[th] order dark fringe

$${r_m}^2 = \frac{2m.\lambda R}{2\mu} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

From eqn. 5 and 6 we can conclude that the radius of bright and dark rings is proportional to the square root of odd natural numbers and natural numbers respectively.

Corresponding diameter the equations are given by

$$\text{For bright rings} \quad {D_m}^2 = \frac{2(2m+1)\lambda R}{\mu} \ldots\ldots\ldots\ldots \text{(7)}$$

$$\text{For dark rings} \quad {D_m}^2 = \frac{4m.\lambda R}{\mu} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(8)}$$

- **<u>Central Fringe</u>:**
  At the point of contact of lens and glass plate d=0. So from equation (3) the condition for destructive interference will be satisfied with m=0.This indicates that the central fringe is dark and appears as dark spot.

- **Determination of Wavelength(λ):**
Newton's ring experiment can be used to determine the wavelength of monochromatic light. The diameter of m^{th} order bright or dark ring from Eq. (7) and Eq. (8) for an air film (μ=1)

For bright rings $\mathbf{D_m}^2 = 2(2m+1)\lambda R \ldots\ldots\ldots\ldots (9)$

For dark rings $\mathbf{D_m}^2 = \mathbf{4m.\lambda R}$

If $D_{(m+p)}$ is the diameter of the $(m + p)$ th bright ring, then

$$\mathbf{D_{m+p}}^2 = 2[2(m+p)+1]\lambda R \ldots\ldots\ldots\ldots\ldots\ldots \textbf{(10)}$$

Subtracting Eq. 9 from Eq. 10 we get

$$\mathbf{D_{m+p}}^2 - \mathbf{D_m}^2 = 2[2(m+p)+1]\lambda R - 2(2m+1)\lambda R$$
$$= 4p\lambda R$$

Therefore,

$$\lambda = \frac{\mathbf{D_{m+p}}^2 - \mathbf{D_m}^2}{4p\lambda R}$$

- **Determination of refractive index of liquid:**
To determine the refractive index of liquid we can use Newton's ring. At first the diameter of m^{th} and $(m+p)^{th}$ bright or dark rings are measured with air film. Then the diameters of these rings are measured again by forming a liquid film in between the convex surface and the glass plate. After replacing the air with liquid the diameter of the rings are decreased.

For air film $(\mathbf{D_{m+p}}^2 - \mathbf{D_m}^2)_{air} = 4p\lambda R$

For liquid film $(\mathbf{D_{m+p}}^2 - \mathbf{D_m}^2)_{liqiud} = \frac{4p\lambda R}{\mu}$

Therefore, $\mu = \frac{(\mathbf{D_{m+p}}^2 - \mathbf{D_m}^2)_{air}}{(\mathbf{D_{m+p}}^2 - \mathbf{D_m}^2)_{liqiud}}$

B. Newton's ring with transmitted light:

Newton's ring can also be observed with transmitted light**.** There are two differences in the reflected and transmitted systems of rings-

(i) The rings observed in transmitted light are exactly complementary to those seen in the reflected light, so that the central spot is now bright.

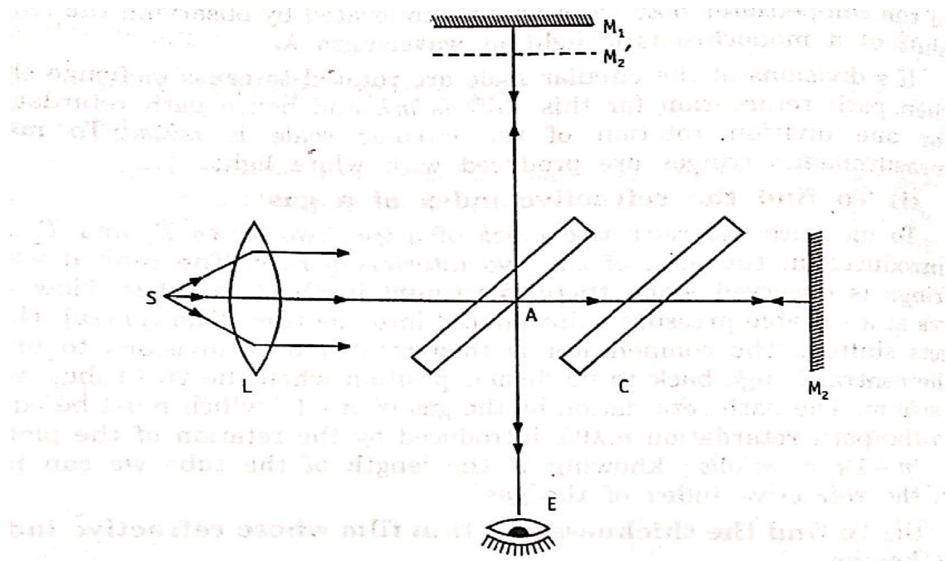
(ii) (ii) The rings in transmitted light are much poorer in contrast than those in reflected light.

Interferometer

An interferometer is an arrangement for obtaining interference effect with a large path difference between the two interfering rays.

- **Michelson Interferometer:**

The essential parts of Michelson Interferometer are shown schematically.



- S is a monochromatic source of light and L is a convex lens.
- M_1 and M_2 are two optically plane mirrors highly silvered on the front surface.
- Usually M_2 is fixed and M_1 can be moved accurately by a calibrated screw.
- M_1 and M_2 are also capable of slight rotation about their horizontal as well as vertical axis with the help of screws.

- A is the beam splitter, is a plane glass plate slightly silvered on one side. C is the compensator plane glass plate of the same thickness as A. Both are mounted vertically and at an angle 45° to the direction of the incident light.

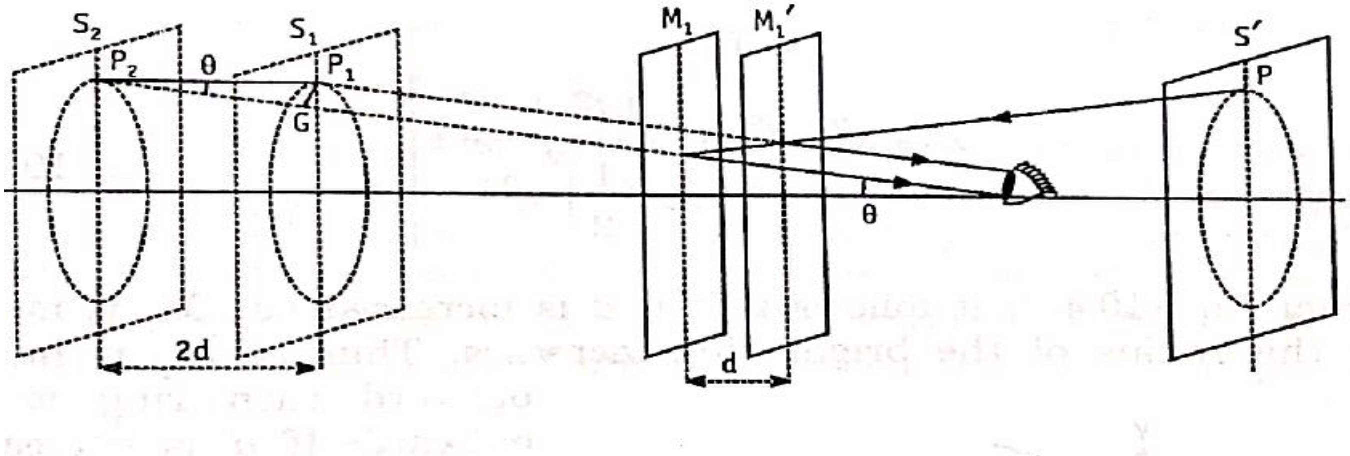
- **Formation of fringes by Michelson interferometer:**

  - Light from an extended light source S is made parallel by the lens L and allowed to fall on A. The light is divided into two parts of equal amplitudes by partial reflection and transmission at the rear side of A. The transmitted light proceeds to M_2 and the reflected light proceeds to M_1. After returning from M_2 the light is reflected at the back of A and proceeds towards AE. The light after reflected from M_1 passes through A to reach the eye E. As the light waves are coherent, they produce interference fringes which can be seen by looking from E into the mirror M_1.

  - Light wave from M_1 cross the glass plate A thrice whereas reflected light wave from M_2 traverses A only once. To compensate for this extra path for glass an exactly similar glass plate C is used.

  - One of the interfering beams is coming from M_1 and the other appears to come by reflection from M_2'. M_2' is basically the virtual image of M_2. Depending on the separation (d) between M_1 and M_2' and the angle between the surfaces, we may get fringes of different shapes (Circular, Straight, and Curved.)

- **Formation of Circular Fringes**:
When the surfaces of M_1 and M_2 are perfectly vertical and at a right angle to each other, the image M_2' will be parallel to M_1. In these condition concentric circular fringes are formed.

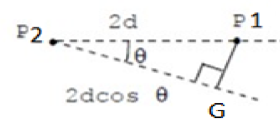


From the above figure the two virtual coherent sources S_1 and S_2 are the images of S' formed in M_2' and M_1. P_1 and P_2 are the two virtual images of the point P on S' formed by the two mirrors M_1 and M_2'.

If θ be the angle of inclination of the reflected rays with the normal, then

the path difference P_2G between the two rays coming from P1 and P2

is $2dcos\theta$. They rays will reinforce each other to produce maximum intensity when

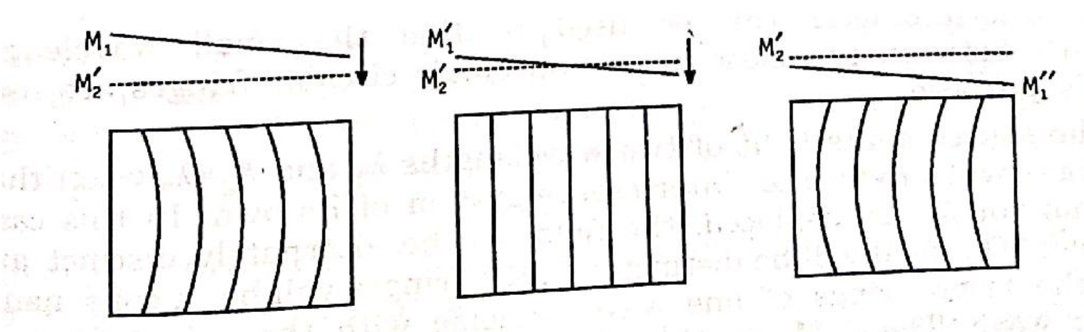


$$2dcos\theta = m\lambda$$

Where d and λ are constants, so θ will be constant for given order number (m). Hence maxima will be in the form of concentric circles about the foot of the perpendicular from the eyes to the mirror as a common centre.

- This type of fringes are called fringe of equal inclination.
- Fringes are non-localized and situated at infinity.

- **Formation of Localized fringes**:
When the mirror M_1 and the virtual image M_2' of M_2 are not exactly parallel localized fringes are produced.
  - **Shapes of the Localized fringes**:



- **Application of Michelson Interferometer**:

  - **Determination of Wavelength of a monochromatic light**:
    For this purpose the interferometer is adjusted to obtain circular fringes in the field of view of the observing telescope. Then the mirror M_1 is through a distance $\lambda/2$.
    The path difference will be changed by $2\times \lambda/2= \lambda$ and hence the position of a bright fringe is taken by the next bright one.
    Let, position of M_1 is shifted by a distance x until m bright fringes cross the cross-wire of the observing telescope.

$$\text{Therefore, } \boldsymbol{x = m\lambda/2}$$
$$\boldsymbol{\lambda = 2x/m}$$

    Now, x can be measured with the help of a micrometer screw. Thus by counting m we can find out λ.

  - **Determination of difference in Wavelength:**
    If the source emits the light of two wavelengths λ_1 and λ_2 ($\lambda_1 > \lambda_2$) then each wave will produce an interference system of its own. In this situation if M_1 is displaced, the field will be alternately distinct and indistinct. The fringes will be in consonance when the bright rings of one wave coincide with the bright ring of another. Similarly the fringes will be in dissonance bright ring of one wave coincide with the dark ring of another.
    Let mirror M_1 is displaced by a distance d so that the fringes pass from one consonance to next consonance through the intermediate state of dissonance. This will happen when value of d is such that

$$\boldsymbol{d = \frac{m\lambda_1}{2} = \frac{(m+1)\lambda_2}{2}}$$

    Where m and (m+1) represents the number of fringe shift for the light of wavelength λ_1 and λ_2.

$$or, \qquad \boldsymbol{\frac{2d}{\lambda_1} - \frac{2d}{\lambda_2} = 1}$$

    If λ_1 is known, we can find λ_2 from the above relation. Then difference in wavelength can be determined.

  - **Determination of refractive index of a material:**
    To determine the refractive index of a material (μ), the interferometer is first to be adjusted for white light fringes when the optical path for two interfering beam are made equal. A thin wire is attached to the middle of the mirror M_1 and the central achromatic fringe with white light is to be made coincident with the wire.
    Now a thin plate ($refractive\ index = \mu\ and\ thickness = t$) is introduced in the path of one of the interfering rays. An extra optical path $(\mu - 1)t$ is introduced in the side of the plate.
    Since the ray travels twice though the plate, the path difference introduced is $2(\mu - 1)t$ between the two interfering beam.
    Due to this extra path central fringe will be displaced from the wire. The mirror M_1 is then to be displaced d until the central fringe again coincides with the wire. In that case

$$\boldsymbol{2d = 2(\mu - 1)t}$$
$$\boldsymbol{or, \qquad d = (\mu - 1)t}$$

    If thickness of the plate t is known, we can find μ from the above relation.

- **References**:
  1. A Textbook of Optics -N. Subrahmanyam, Brij Lal.
  2. A Textbook on Light-B. Ghosh, K.G. Mazumdar.