Fresnel's Diffraction

- **Fresnel's assumption's :**

Fresnel gave a satisfactory explanation of this phenomenon by using Huygen's principle in conjunction with the principle of super-position.
According to Huygen's principle each point on the wavefront acts as the source of the secondary waves. The mutual interference of these secondary waves derived from a particular wavefront, produces the phenomenon of diffraction.

- **properties of Fresnel's Diffraction :**

i) Source and the screen are at finite distance from the obstacle/Aperture.

ii) spherical / cylindrical wavefront falls on the obstacle/aperture.

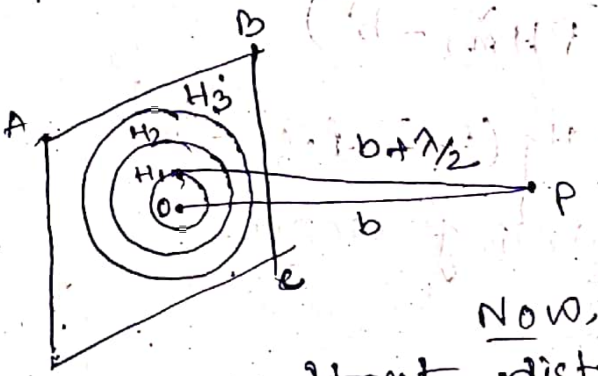
iii) No lenses are used in Fresnel Diffraction.

iv) Waves falling on the obstacle/Aperture will not be in the same phase.

v) Fresnel diffraction is the general case of diffraction, which reduces to Frauenhoffer case when the source and the screen are at infinite distance from the obstacle/Aperture.

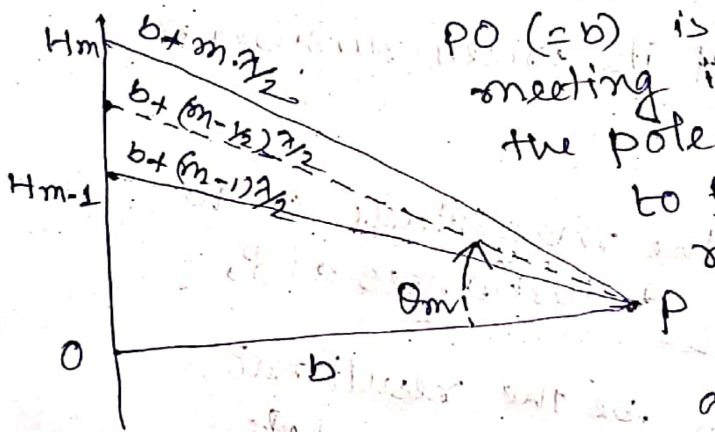
Fresnel's half-period Zones of a plane wavefront and their applications.......

... The phenomena ~~based~~ of diffraction of light on the basis of the mutual interference of the secondary waves or wavelets from the various points of a wavefront.



Let ABCD be the plane wavefront of light of wavelength λ, ~~advancing to the right~~

Now, we have to find out the resultant disturbance at P due all the wavelets coming from every points of the wavefront, the whole wavefront is divided into a Fresnel half-period zones in the following way.



Now at P, a perpendicular PO ($=b$) is drawn on the wavefront meeting it at O which is called the pole of the wave with respect to P. With P as centre and radii $(b+\lambda/2)$, $(b+\frac{2\lambda}{2})$, $(b+\frac{3\lambda}{2})$ ... etc. spheres are drawn the sections of which by the plane of the wavefront are concentric circles H_1, H_2, H_3 etc. The area enclosed by the circle H_1 is called first half period zone. The annular zone between the circles H_1 and H_2 is called second half-period zone, and so on.

- Area of Zones :

Now the area of the m th zone is the area in between the circles H_m and H_{m-1} is,

$$A_m = \pi(PH_m^2 - b^2) - \pi(PH_{m-1}^2 - b^2)$$

$$= \pi\left\{\left(b + m\cdot\frac{\lambda}{2}\right)^2 - b^2\right\} - \pi\left\{\left(b + \overline{m-1}\cdot\frac{\lambda}{2}\right)^2 - b^2\right\}$$

Now, as $b >> \lambda$ so negleting we get.

$$= \pi b\lambda + \pi(2m-1)\frac{\lambda^2}{4}$$

$$\simeq \pi b\lambda \quad [b >> \lambda]$$

So, as $b >> \lambda$, mostly all of the zones are approximately of equal area.

- Actually the area of a zones increases with the increase of its order number m

- - Factors governing the magnitude of the of the amplitude of disturbances at P, :

Let $a_1, a_2, a_3 \ldots a_m$ be the resultant amplitudes at P due to all the wavelets comming from 1st, 2nd, 3rd ... m th zones respectively,

... hence the resultant amplitude at P due to all the zone is,

$$R = a_1 + a_2 + a_3 + \cdots + a_m$$

According to Fresnel the amplitude a_m at P due to wavelets coming from the mth zone depends on the following factors -

i) $a_m \propto A_m$ (Area of m th zone)

ii) $a_m \propto \frac{1}{d_m}$ (d_m distance of P from the mth zone)

iii) $a_m \propto$ obliquity factor $f(\theta_m)$.

where θ_m is the angle which the diffraction of P from the m th zone makes with OP. the factor $f(\theta_m)$ is 1, for $\theta_m = 0$ and

$f(\theta_m) = 0$, when $\theta_m = 90$

Since $\frac{A_m}{d_m} = \pi\lambda$ (constant), the successive amplitudes a_1, a_2, a_3 etc. will be decending order of magnitude due to obliquity factor.

- phase consideration :

As per a given zone is $\lambda/2$ farther away from P than the previous one, the disturbances from alternate zones will have opposite phase at P. So,

$$R = a_1 - a_2 + a_3 - a_4 + \cdots + (-1)^{m-1} a_m$$

So, it can be written as,

$$R = \frac{a_1}{2} + \left(\frac{a_1 + a_3}{2} - a_2\right) + \left(\frac{a_3 + a_5}{2} - a_4\right) + \cdots + \frac{a_m}{2}$$

[if m is odd]

$$= \frac{a_1}{2} + \left(\frac{a_1 + a_3}{2} - a_2\right) + \left(\frac{a_3 + a_5}{2} - a_4\right) + \cdots + \frac{a_{m-1}}{2} - a_m$$

[if m is even]

as a_1, a_2, a_3 are in descending order of magnitudes $\frac{a_1 + a_3}{2} \approx a_2$ and so on.
So, all the terms within the bracket in last two eqn cancel out and we get

$$R = \frac{a_1}{2} + \frac{a_m}{2} \quad (m = \text{odd})$$

$$R = \frac{a_1}{2} + \frac{a_{m-1}}{2} - a_m \quad (m = \text{even})$$

Now, when m is very large, greater obliquity of zones causes a_{m-1} and a_m vanishes and thus $R \simeq \frac{a_1}{2}$

So, the resultant amplitude at P due to the whole wavefront is equal to half the amplitude of the secondary waves from the 1st half period zone.

Applications :

a. Rectilinear propagation of light
b. Circular disc in the path of a plane wavefront.
c. circular aperture in the path of a plane wavefront.
d. Absence of reverse wave in Hygen's principle.

Scanned with CamScanner

Applications :

(a) Explanation of rectilinear propagation of light :

Suppose a small circular opaque obstacle is placed at O which covers only the first half-period zone of the wave [Fig. 11.2-1(a)]. The resultant intensity at P due to the exposed wavefront is evidently proportional to $(a_2/2)^2$. Similarly, if the size of the circular obstacle increases and successively covers the first two, first three, etc. half-period zones, the resultant intensity at P becomes respectively proportional to $(a_3/2)^2$, $(a_4/2)^2$, etc. Thus the illumination at P gradually diminishes and ultimately becomes too small when the size of the obstacle is large enough to intercept an appreciable number of half-period zones. As the sizes of these half-period zones are very small, a tiny obstacle is sufficient to cover a large number of half-period zones by which the light from the source is practically cut off. This fact is interpreted as the rectilinear propagation of light.

(b) Circular disc in the path of a plane wavefront :

Let AB be an opaque circular disc on which the plane waves of a light of wavelength λ are incident in a direction normal to the disc (Fig. 11.2-2). Let us now proceed to find the illumination at points P, P_1, P_2, etc. on the axis of the disc.

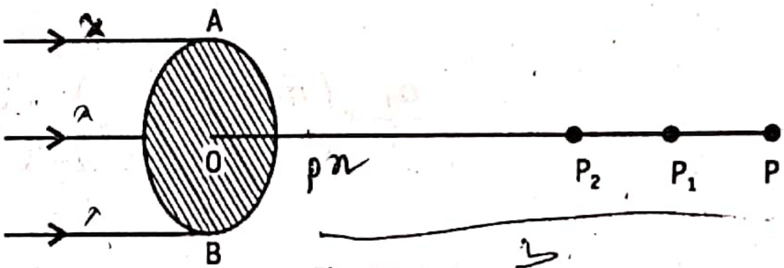


Fig. 11.2-2

The area of a half-period zone with respect to an axial point situated at distance b from the disc, is $\pi b\lambda$. Thus the disc will intercept more number of zones for a nearer point (b less) and for a light of shorter wavelength.

Suppose the point P is at a sufficient distance from the disc for which the size of the first half-period zone is such that a portion of it is only covered by the disc. The illumination at that point is the same as that obtained when the disc is absent. Let now the points of observation be shifted to P_1, P_2, etc., which are nearer to the disc such that the disc respectively intercepts the first, the first two, etc. half-period zones for these points. The resultant intensity at the points P_1, P_2, etc. will then be proportional to $(a_2/2)^2$, $(a_3/2)^2$, etc. As a_1, a_2, a_3, etc. are in the descending order of magnitudes, the intensity at the centre of the shadow of the circular disc gradually decreases. When the point of observation is very close to the disc, total darkness would be obtained.

If the distance b of the point be kept fixed but the size of the disc be increased gradually, the intensity at the point will be proportional to $(a_2/2)^2$, $(a_3/2)^2$, etc. according as the size of disc is such as to cover the first, the first two etc. half-period zones respectively. When the size of the disc is sufficiently large to cover an appreciable number of half-period zones from the first, the point will be totally dark.

Effect of white light:

If white light is employed, then for a given point on the axis, the disc will cover more number of half-period zones for violet light than for red light. Thus the intensity at the point will be less for violet light than for red light causing red colour more prominent than violet colour.

(c) Circular aperture in the path of a plane wavefront:

Let plane waves of a light of wavelength λ, be allowed to pass through a small circular aperture in a screen in a direction at right angles to the screen (Fig. 11.2-3).

The intensity at any point on the axis may be obtained by dividing the aperture into a number of half-period zones with respect to the

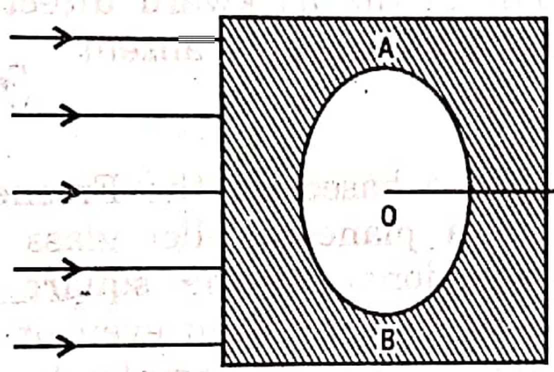


Fig. 11.2-3

given point. The area $\pi b\lambda$ of a zone will be smaller, when the distance b of the point from the aperture is smaller and the wavelength of the light is shorter.

Suppose for a distant point P on the axis, the aperture transmits only the first half-period zone of the wave. The intensity at P will then be proportional to a_1^2, which is four times that obtained with the whole wave.

With a wider aperture or for a nearer point P_1 on the axis, let the aperture transmit only the first two half-period zones of the wave. The resultant intensity at P_1 will be proportional to $(a_1 - a_2)^2$, which is very nearly zero.

For a still nearer point P_2 on the axis, let the aperture transmit only the first three half-period zones of the wave. The resultant intensity at P_2 will then be proportional to $(a_1 - a_2 + a_3)^2$ or very nearly proportional to a_1^2, which is again maximum.

In general, we may say that a point on the axis will have maximum or minimum illumination according as the aperture transmits odd or even number of half-period zones with respect to that point.

If the illumination at the points other than the centre be calculated, we find that round the centre there are alternately bright and dark rings.

Effect of white light :

If white light be employed, then for a given point on the axis, the aperture may contain odd number of half-period zones for a light of one wavelength and even number of half-period zones for a light of another wavelength causing one colour more prominent than the other and we get coloured rings.

(d) Absence of reverse wave in Huygens' principle :

Fresnel assumed the obliquity factor to have the form $f(\theta_m) = (1 + \cos\theta_m)$. For waves travelling along backward direction from the first half-period zone $\theta_m = \pi$ and hence $f(\theta_m) = 0$. Thus the resultant amplitude $R = a_1/2$ at any point in the backward direction would be zero. This means that the backward wave is absent.