Q. Fraunhofer double slit diffraction pattern is observed in the focal plane of a lens of focal length 0.5 m. The wavelength of incident light is 500 nm. The distance between two maxima adjacent to the maximum of zero order is 5 mm and the fourth order maximum is missing. Find the width of each slit and the distance between their centres.

Ans: As for missing of 4th order maximum we have,

$$\boxed{\frac{a+b}{a} = 4}$$

$$\Rightarrow b = 3a$$

Now, for 1st order maximum,

$$(a+b)\sin\theta_1 = \lambda$$

as, θ_1 is small $\rightarrow \sin\theta_1 \approx \theta_1 \Rightarrow \theta_1 \approx \frac{\lambda}{a+b} = \frac{\lambda}{4a}$.

∴ Distance between two maxima adjacent to the maximum of zero order is,

$$x = f \times 2\theta_1 = \frac{2f\lambda}{4a}$$

$$\Rightarrow a = \frac{f\lambda}{2x} = \frac{0.5 \times 500 \times 10^{-9}}{2 \times 5 \times 10^{-3}}\ m = 0.025\ mm$$

So, $b = 3a = 0.075$ mm.

Q: A diffraction grating 2 cm wide is just able to resolve sodium D-lines in second order. Find the number of rullings per mm.

Ans: we know $\boxed{\frac{\lambda}{d\lambda} = N.m}$

Now, $m = 2$, $\lambda = \frac{589 + 589.6}{2}\ nm = 589.3\ nm$

$d\lambda = 0.6\ nm$

Therefore, $N = \frac{\lambda}{m.d\lambda} = \frac{589.3}{2 \times 0.6} = 491$

∴ No. of rullings per mm $= \frac{\text{Total no. of lines}}{\text{width of Grating in mm}} = \frac{491}{20} = 24.5$

Q: A parallel beam of light of wavelength 500 nm is incident normally on a narrow slit of width 0.2 mm. The Fraunhoffer diffraction is observed on a screen which is placed at the focal plane of a convex lens of focal length 20 cm. Calculate the approximate distance between first two maxima

Ans: We know that the 1st two maxima occour at,

(From Theory) →

$$\alpha_1 \left(\approx 3.\frac{\pi}{2}\right) = 1.43\pi$$
$$\alpha_2 \left(\approx 5.\frac{\pi}{2}\right) = 2.46\pi$$

So, corresponding angles of diffraction are given by.

$$a \sin\theta_1 = 1.43\lambda$$
$$a \sin\theta_2 = 2.46\lambda$$

$$\therefore \theta_1 = \sin^{-1}\left[\frac{1.43 \times 500 \times 10^{-9}}{0.2 \times 10^{-3}}\right] = 3.57 \times 10^{-3} \text{ rad}$$

Similarly $\theta_2 = \sin^{-1}[\quad] = 6.15 \times 10^{-3}$ rad.

∴ The linear distance between the first two maximar is. $f \times (\theta_2 - \theta_1) = 0.516$ mm

Q: The objective of Telescope has a diameter 2.54 m Assuming the mean wavelength of incident light to be 550 nm. Calculate the least angular separation of two stars which can be resolved by it.

Ans: The least resolvable angle = $\theta = \frac{1.22\lambda}{D}$ radian

Now, $D = 2.54$ m, $\lambda = 550 \times 10^{-9}$ m,

$$\theta = \frac{1.22 \times 550 \times 10^{-9}}{2.54} = 2.642 \times 10^{-7} \text{ radian.}$$

$$\theta = \left(2.642 \times 10^{-7} \times \frac{180}{\pi} \times 3600\right)''$$

$$= (\quad)''$$

Q: Consider a grating of width 5 cm with slits of width $a = 0.001$ mm separated by a distance of 0.002 mm. How many orders would be visible at $\lambda = 550$ nm? Calculate the width of principle maxima. Would there be any missing orders?

Ans:

For Grating $\boxed{(a+b)\sin\theta = m\lambda}$

Here, $a = 0.001$ mm

$b = 0.002$ "

$\lambda = 550 \times 10^{-9}$ mm $= 550 \times 10^{-6}$ mm

Now, for $\theta = 90°$

$$m = \frac{a+b}{\lambda} = \frac{0.003}{550 \times 10^{-6}} = 5.45$$

So, maximum No. of observable orders = 5

Theory → According to Resolving power of a Grating

Angular width of principle maxima $= 2d\theta$

$$\boxed{2d\theta = \frac{2\lambda}{N(a+b)\cos\theta}}$$

$$= \frac{2 \times 550 \times 10^{-9}}{5 \times 10^{-2}}$$

$$2d\theta = 2.2 \times 10^{-5} \text{ rad.}$$

For maximum $\cos\theta \approx 1$

$N(a+b)$ = width of Grating $= 5 \times 10^{-2}$ m

Now, $\frac{a+b}{a} = \frac{3}{1}$

This indicates that 3rd, 6th, 9th etc order maxima will be missing.

Q. An oil immersion microscope just resolves the rullings of a grating having 3900 lines per mm when a light of wavelength 400 nm is employed. Find the Numerical Aperture (N.A.) (2)

Ans: So, the distance between two rullings $= \frac{1}{3900}$ mm.

thus, $\boxed{\frac{1}{3900}\text{ mm} = \frac{1.22\lambda}{2(N.A)}}$

$N.A = 0.95$

Q. The refractive indices of a glass for wavelengths 656.3 nm and 527.0 nm are respectively 1.6545 and 1.6635. Calculate the length of the base of a 60° prism of this glass which can just resolve sodium lines of wavelengths 589 nm & 589.6nm

Ans: The change of r.i = $d\mu$

" " of wavelength = $d\lambda$

Now, $\boxed{t\frac{d\mu}{d\lambda} = \frac{\lambda}{d\lambda}}$

$\frac{d\mu}{d\lambda} = \frac{1.6635 - 6.6545}{(656.3 - 527.0)\times 10^{-9}} = 69605\ m^{-1}$

$\lambda = \frac{589 + 589.6}{2}$ nm $= 589.3$ nm

$d\lambda = 0.6$ nm

$t = \frac{\lambda/d\lambda}{d\mu/d\lambda} = \frac{589.3}{0.6} \times \frac{1}{69605} \simeq 1.4$ cm

Assignment 5×4 = 20

Q. 1. A convex lens of focal length 40 cm is employed to focus the Fraunhofer diffraction pattern of a single slit of 0.3 mm width. Calculate the linear distance of first order dark band from the central band: take the wavelength of light $\lambda = 589$ nm. [4]

H.T.

Q. 2. How many orders would be visible, if the wavelength of incident light is 589 nm and the number of lines in the grating is 104 per mm? [4]

H.T.

Q. 3. Sodium light of wavelengths 589.0 ~~mm~~ nm and 589.6 nm are made incident normally on a grating having 500 lines per mm. Calculate the angular dispersion of these lines in the spectrum of first order. [4]

H.T.

Q. 4. Calculate the least width that a grating must have to resolve two components of sodium D lines (having wavelength 589 nm and 589.6 nm) in the second order. The number of lines per mm of the grating is 80. [4]

H.T.

Q. Two stars situated at a distance of 9.5×10^{12} Km from a telescope of diameter 20 cm are sending light of wavelength 600 nm. Find the distance of separation of the stars for ~~what~~ which they are just resolved. [4]