

DSE 3T

PHYSICS (H), 6<sup>TH</sup> SEM

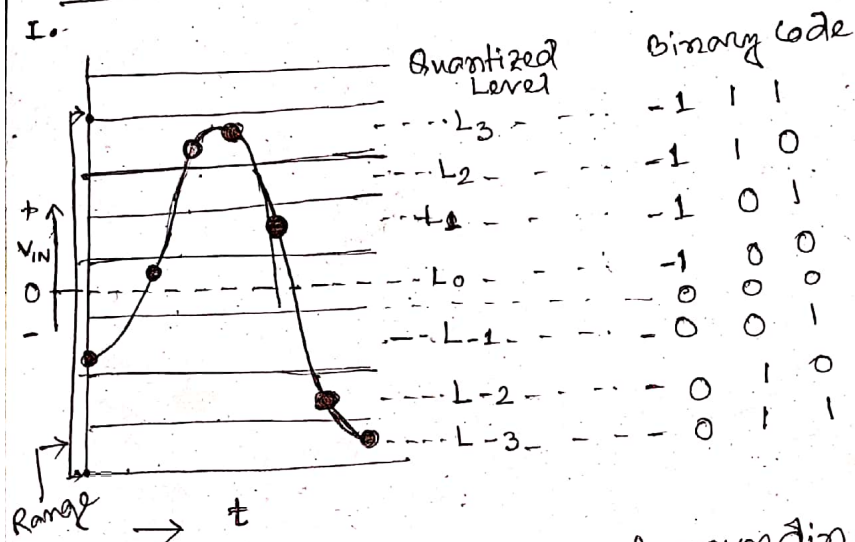
PAPER: Communication Electronics

TOPIC: DIGITAL PULSE MODULATION

## PCM (Pulse Code Modulation)

The basic elements of PCM can now now be put in place. The analog signal  $v_m(t)$  passes through an antialiasing filter. The float-top samples are converted to digital numbers through an analog to digital converter. So, for A/D conversion one important step is now Quantization.

### Quantization of PCM :



Quantization is the process of rounding off the values of the float-top samples to certain predetermined levels in order to make a finite and manageable number of levels available to A/D converter. Otherwise the sampling levels could take any value of the peak to peak range of the analog signal, which may result an infinite number of levels.

In the Quantization process total signal range is divided into a number of subranges (fig). Each subrange has its mid value designated as the standard or code level for that range.

Comparators are used to determine which sub-range a given pulse amplitude is in, and code for that sub-range is generated.

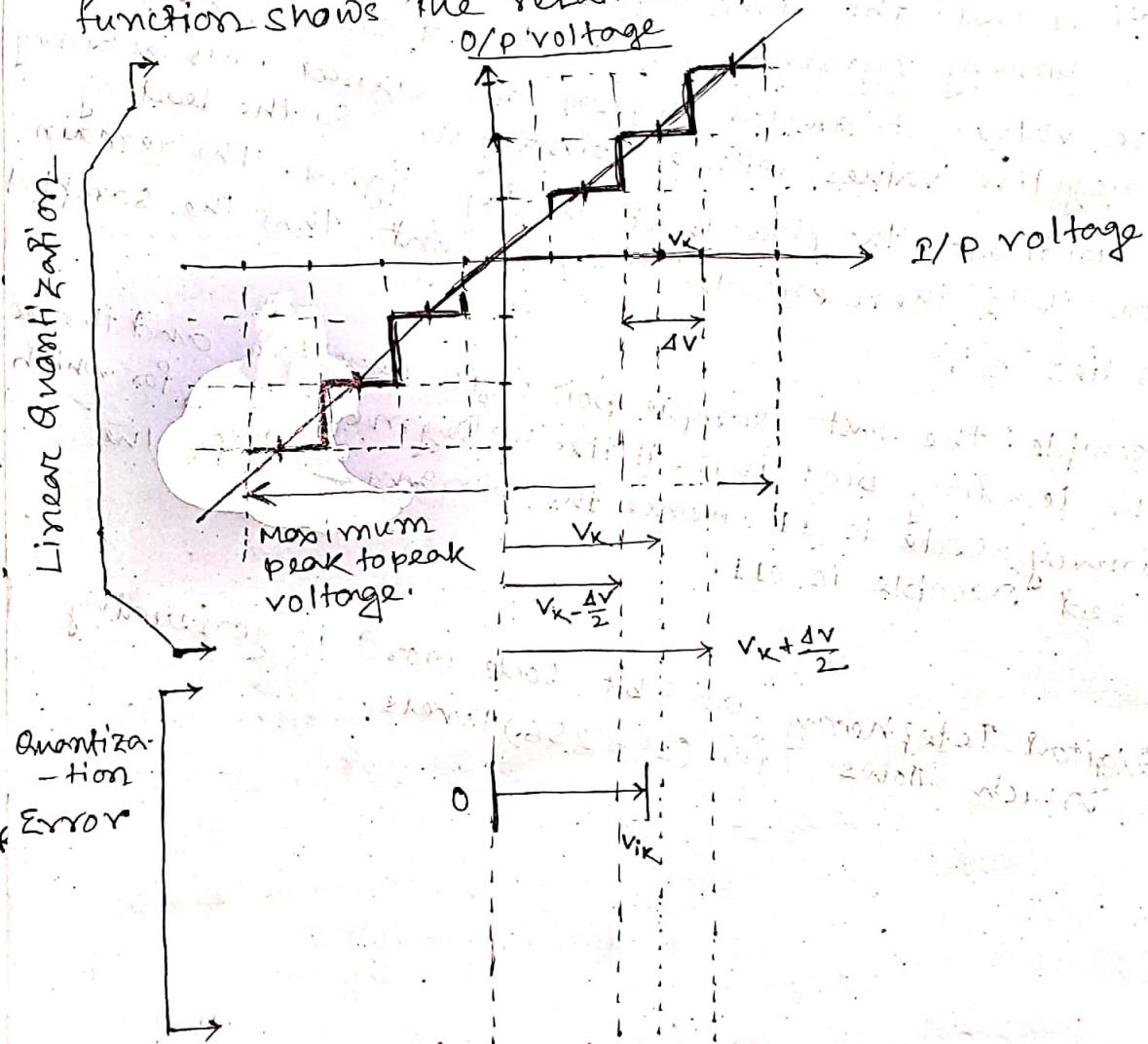
To illustrate that process a small no. of levels, eight in all, are shown, along with a binary code for each level. It will be noticed that the level representing zero analog volts has two binary numbers, one for 0+ and one for 0-. Positive values of analog signal are signed with a binary 1 & negative values with a binary 0. So the leading beats indicates the polarity of analog signal. The remaining two bits then encode the segment that the sampled value lies in.

For example; the last sample point is negative and therefore the leading beat is 0, it lies in segment L-3 for which the binary code is 11, hence the binary code for the quantized sample is 011.

\* In digital Telephony, an 8 bit code word is generally used, which allows for  $(2^8 = 256)$  levels.



Ⓜ Now, the Quantization can represent in different way. The straight line shows the linear I/P-O/P relationship that would exist if the quantization would not developed (employed). Mainly staircase function shows the relationship of quantization.



... This particular function is referred to as mid-tread quantization, since the quantization levels correspond to the input values at the middle of the tread on the staircase function. It is also possible to have a mid rise type of function.

Ⓚ

\* Quantization Error : It is the function of the I/P voltage

This is the difference between the quantized level & analog input [i.e.  $V_k - v_{ik}$ ]. Quantization error arrives as a noise referred to quant... Noise, on the analog signal when it is recovered at the receiver. As can be seen, the quantization error can lie between  $\pm \Delta V/2$  & assumed it has a uniform probability density distribution, also known as mean-square quantization error

$$E_{nq}^2 = \frac{(\Delta V)^2}{12}$$

— This is also

the mean-square quantization noise voltage. For total No. of  $L$  levels, p-p. signal range is  $\pm L \Delta V/2$  & for a signal that has a uniform probability density distribution within the range, the mean square-signal voltage is

$$E_s^2 = \frac{[L \Delta V]^2}{12}$$

So, the signal to quantization noise ratio is,

$$\left(\frac{S}{N}\right)_q = \frac{E_s^2}{E_{nq}^2} = L^2$$

\* This shows that to maintain a high  $S/N_q$  ratio the no. of steps should be high, for example, for  $L=256$ ,

$$S/N_q \cong 48 \text{ db}$$

In terms of the number of bits per code word  $n$ ,  $L=2^n$

and hence,  $(S/N)_q = 2^{2n}$



⊛ if  $m(t)$  is the sine wave that occupies the full range, then,

$$\left(\frac{S}{N}\right)_q = 1.5L^2$$

- The ratio between the peak to peak and r.m.s. values of signal voltage will be some value. If the  $K = \frac{E_{r.m.s.}}{E_{max}}$  distortion is to be avoided the maximum peak signal level must not be allowed to exceed half the total i/p voltage range of.

$$E_{max} = L \cdot \frac{4V}{2}$$

Now, the signal to quantization noise ratio in that case is,

$$\begin{aligned} \left(\frac{S}{N}\right)_q &= \frac{E_{r.m.s.}^2}{E_{ny}^2} \\ &= \left[\frac{K L \frac{4V}{2}}{2}\right]^2 \cdot \frac{12}{(4V)^2} \\ &= 3K^2 L^2 \end{aligned}$$

Eg: A PCM system is to have a signal to noise ratio of 40 dB. The signals are speech and an r.m.s to peak ratio of -10 dB is allowed for. Find the number of bits per code word required.

$$\rightarrow \text{Let } \left(\frac{S}{N}\right)_q = 3K^2 L^2 = 3K^2 2^{2n}$$

$$\text{So, } 10 \log \left(\frac{S}{N}\right)_q = 10 \log 3 + 20 \log K + 20n \log 2$$

$$\left(\frac{S}{N}\right)_q \text{ in dB} = 4.77 + (K) \text{ dB} + 6.02n$$

$$\text{So, } 40 = 4.77 - 10 + 6.02n$$

$$n = \frac{40 - 4.77 + 10}{6.02} \approx 7.5 = 8 \text{ (rounded up)} \\ \text{(Ans)}$$

A useful relationship between the bandwidth  $B$ , required to transmit a signal - channel PCM signal and  $(S/N)_q$  becomes,

$$(S/N)_q = 2^{2n}$$

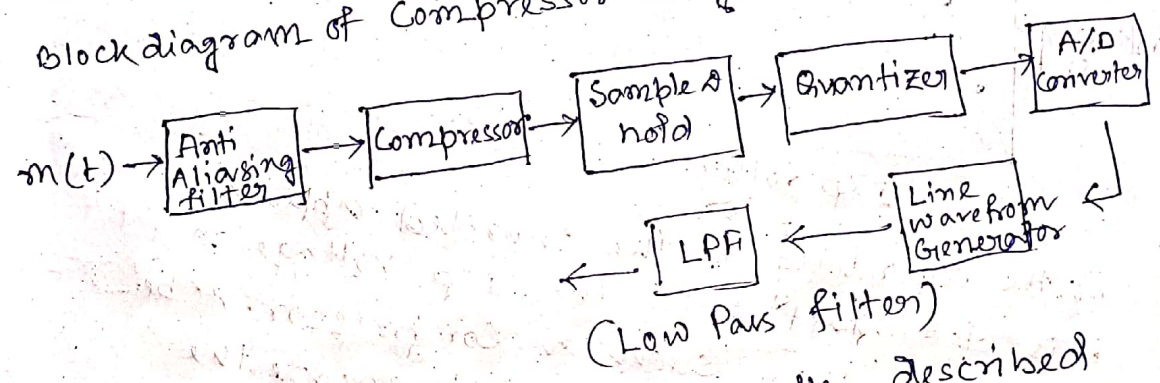
Now, in dB,  $(S/N)_q \text{ in dB} = 10 \log (2^{2n})$

$$\approx 6n$$

Compression of PCM:

With speech it is found that the peaks of the signal only infrequently extend over the full range of the input, most of the time residing within a small range about zero. In effect, the signal does not have a uniform probability density function and as a result the  $(S/N)_q$  ratio is lower than that given by  $(S/N)_q = \frac{E_s}{E_{nq}} = L^2$

\* Block diagram of Compressor stage:



The compression functions are normally described in terms of normalized voltages. Let  $v_i$  represent the i/p voltage and  $v_{i \max}$  its maximum value. Denote that the normalized input voltage by  $x$ , then,

$$x = \frac{v_i}{v_{i \max}}$$

Now, in the similar manner the o/p voltage as

$$y = \frac{v_o}{v_{o \max}}$$

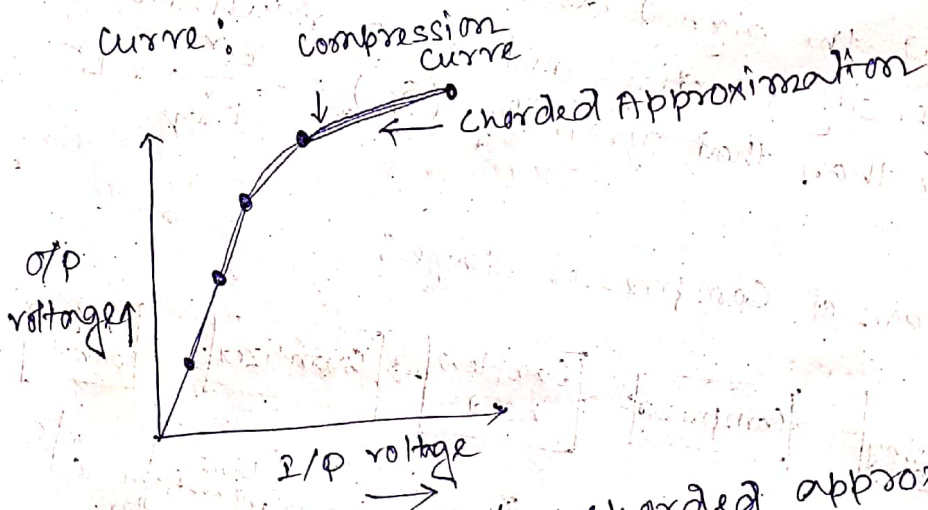


In terms of normalized voltages the  $\mu$  law is described by,

$$y = [\text{sign}(v_i)] \left[ \frac{\ln(1 + \mu|x|)}{\ln(1 + \mu)} \right]$$

- Sign  $v_i \rightarrow$  indicate the polarity of  $v_i$
- $|x|$  is the magnitude of  $x$ .
- $\mu \rightarrow$  Compression parameter, which determines the degree of compression.

• Chorded Approximation to a Compression Curve:



So, according to the chorded approximation it is clear that how the I/P voltage might be quantized by such a compressor. As before the leading bit can be used to encode the analog polarity.



## PCM Receiver:

The receiving section of a codec must provide the inverse operations to those of the transmitter. The function of the I/P filter is to limit the noise bandwidth and to complete the waveform shaping required for the avoidance of ISI. A pulse regenerator is used to generate new pulses that are free of thermal noise, but note that quantization noise is always present and cannot be removed completely.

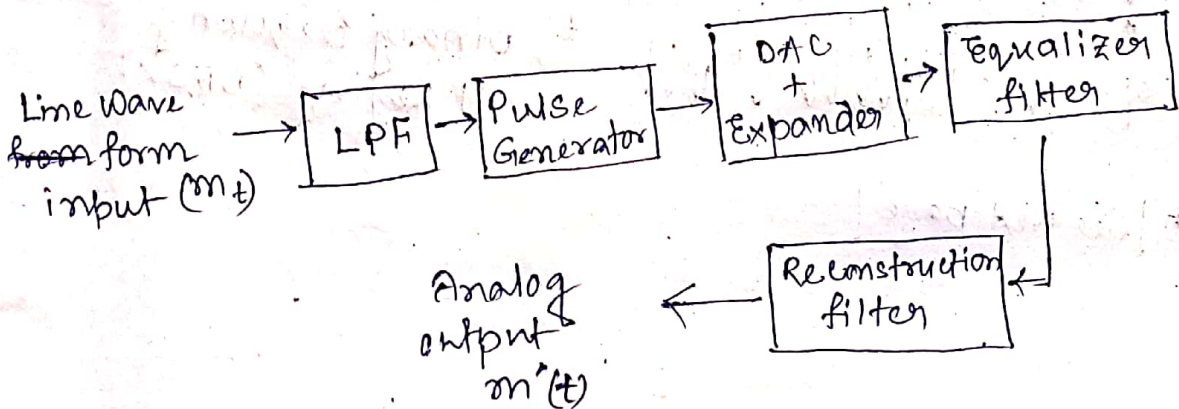
The digital to analog converter (DAC) converts the binary signal into flat-top samples and in the process provides the expansion necessary to compensate for the compression applied at the transmitter.

\*equalizer\* The equalizer\* filter following the DAC compensates the aperture distortion introduced by flat top sampling.

The equalizer is followed by a low-pass filter, often referred to as a reconstruction filter, which essentially recovers the analog signal by passing only the low-frequency part of the spectrum.

\*\* However, quantization noise is also present on the output so that the analog output  $m''(t)$  is not identical to  $m(t)$ .

### • Block diagram of a PCM receiver



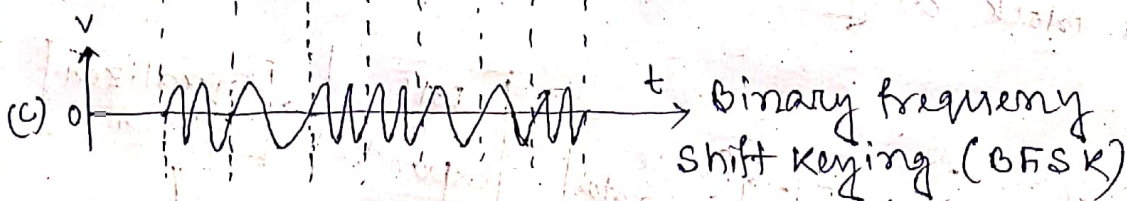
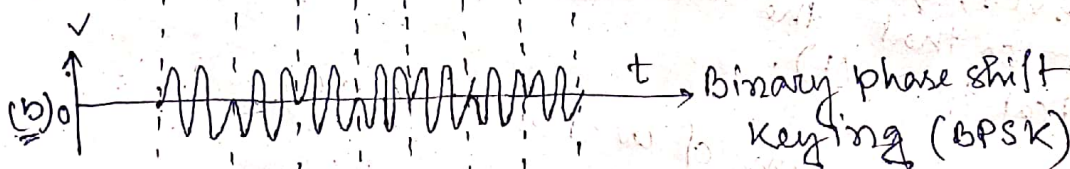
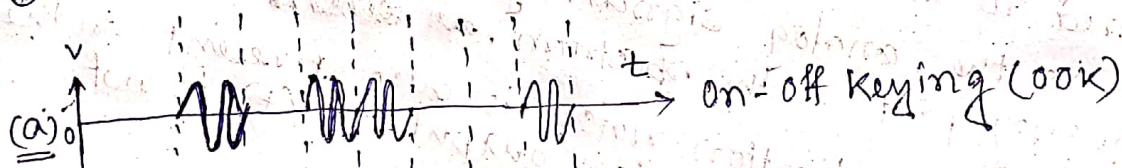
## Digital Carrier Systems :

Digital data may be modulated onto a carrier wave. In general, the function of the carrier is to shift the original digital data from the baseband region into a pass band region of the frequency spectrum. Passband signals are needed for example, in radio transmission & also for frequency multiplexing on lines.

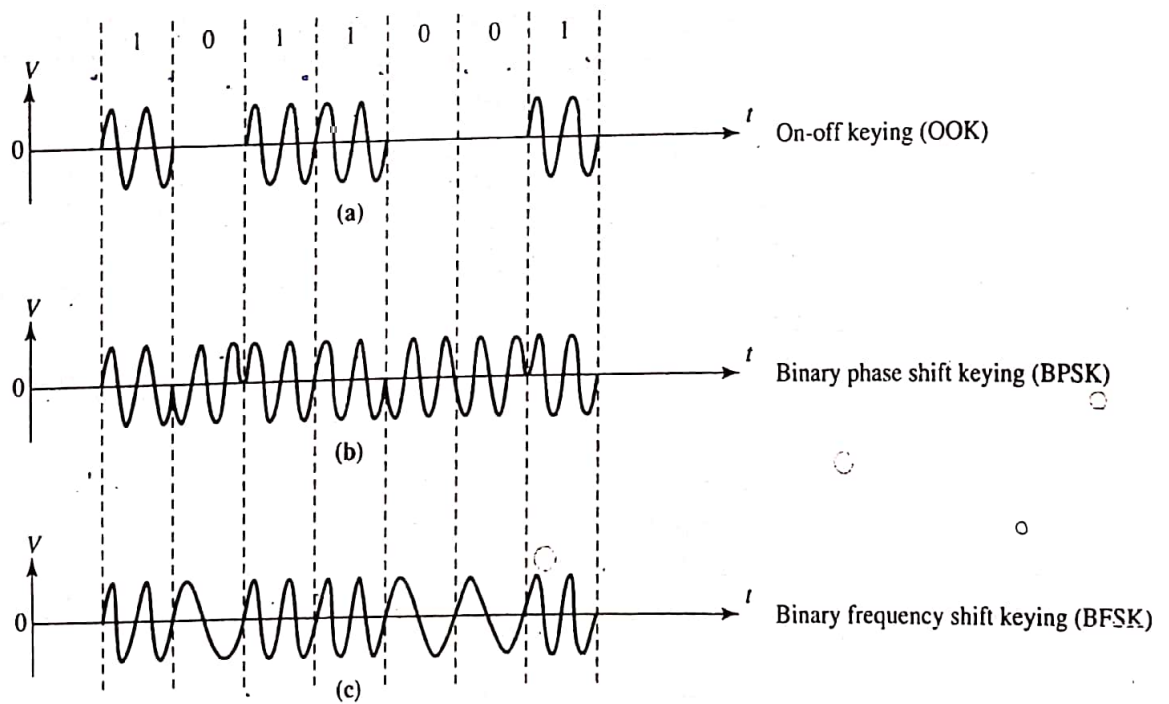
The modulation methods of amplitude, frequency and phase, ~~described in~~ are all feasible for digital signals and in fact are usually easier to implement than is the case with analog signals:

- The average bit energy can be found from the average received power and the bit rate (fig.)

⊗



⊗ [See Next page]



**Figure 12.9.1** Binary modulated carriers: (a) binary amplitude shift keying (BASK), also known as on-off keying (OOK); (b) binary phase shift keying (BPSK), also known as phase reversal keying (PRK); (c) binary frequency shift keying (BFSK).

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☒ If the average received power is the same in each case, the bit energy will also be the same, but this requires that during the on periods for OOK the transmitted power must be doubled or the power amplifiers operated at a peak voltage level that is  $\sqrt{2}$  times that for continuous carrier systems.

If the comparison is made on the basis of the same value of received voltage, then the average power, and hence the bit energy, of the OOK modulation is half that of the other modulation methods.

### Next Syllabus

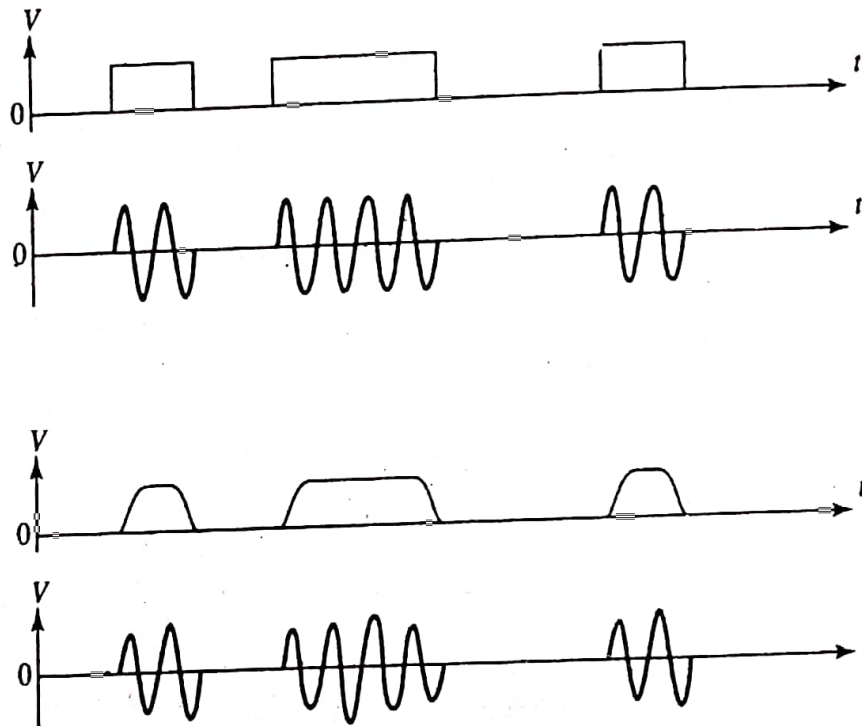
- ASK
- FSK
- PSK
- BPSK



• Completely covered from the book

## Amplitude Shift Keying

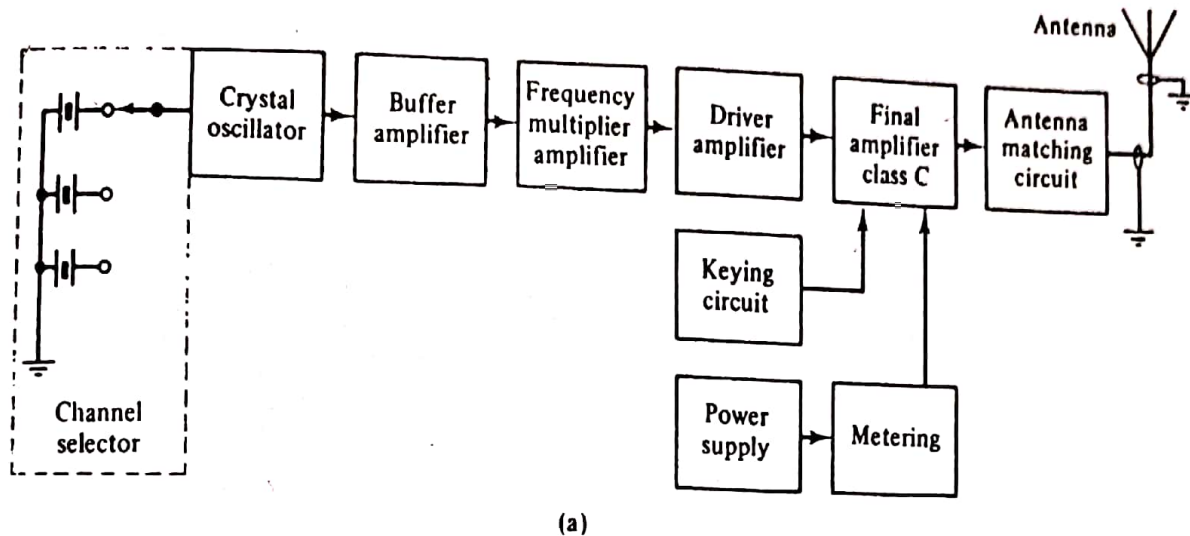
With *amplitude* modulation the digital signal is used to switch the carrier between amplitude levels, and hence it is referred to as *amplitude shift keying* (ASK). The particular case of binary modulation is illustrated in Fig. 12.9.2, where the modulating waveform consists of unipolar pulses. Because in this particular case the carrier is switched on and off, the method is known as *on-off keying* (OOK), or sometimes as *interrupted continuous wave* (ICW) transmission.



**Figure 12.9.2** Amplitude shift keying (ASK) (a) with unipolar rectangular pulses and (b) with filtered pulses.

Amplitude shift keying is fairly simple to implement in practice, but it is less efficient than angle-modulation methods, to be described shortly, and is not as widely used in practice. Applications do arise, however, in such diverse areas as emergency radio transmissions and fiber-optic communications (described in Chapter 20). On-off keying of a radio transmitter may be achieved as shown in the block diagram of Fig. 12.9.3(a)

In Fig. 12.9.3(a), the carrier frequency is generated by a crystal oscillator, which is followed by a buffer amplifier to maintain good frequency stability. Again in the interests of maintaining good frequency stability, the oscillator frequency is usually lower than the required carrier frequency, and one or more frequency multiplier stages are necessary. The driver amplifier is a power amplifier that provides the required drive for the final RF amplifier, which is a class C stage. This is similar to the circuits described in Section 8.10. Although the keying circuit could be used to simply interrupt the current



(a)

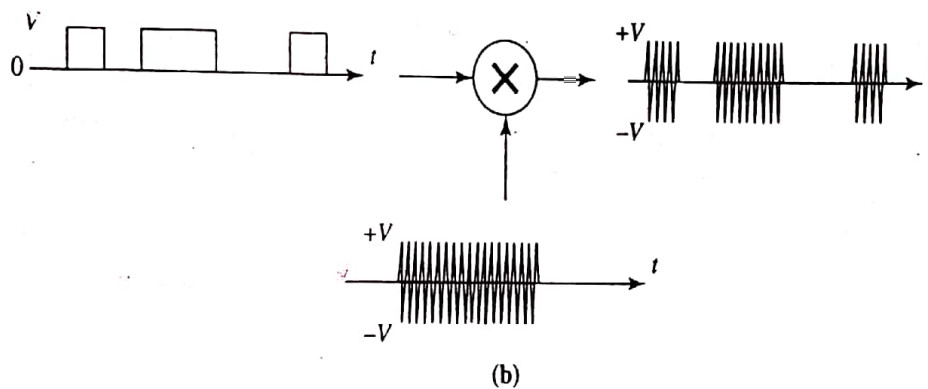


Figure 12.9.3 On-off keying achieved by (a) controlling the bias of a radio telegraph transmitter and (b) use of a multiplier circuit.

in the final amplifier by means of a “make-break” contact, this could give rise to undesirable transients and would be avoided in high-power circuits. The more usual method is to use the keying signal to bias the class C into cutoff for the off binary bits. For radio transmission it is undesirable to have rapid changes in amplitude because these give rise to *sideband spatter*, and the digital modulating waveform is filtered to remove the sharp transitions so that the modulated waveform appears more as shown in Fig. 12.9.1(b). A simple RC filter may be used, and in practice this may have a time constant on the order of 2 ms.

Although keying could take place at a lower power stage in the transmitter, this is not usually done since the class C bias on the final amplifier is derived from the drive signal, and removal of this could result in excessive current rise in the final amplifier. Where only low power stages are involved, a product modulator may be used, as shown in Fig. 12.9.3(b). Denoting the unmodulated carrier by

$$e_c(t) = E_{c \max} \cos(2\pi f_c t + \phi_c) \quad (12.9.1)$$

and the binary modulating waveform as  $e_m(t)$ , then the modulated waveform is

$$e(t) = k e_m(t) \cos(2\pi f_c t + \phi_c) \quad (12.9.2)$$



Although this appears identical to the DSBSC expression given by Eq. (8.9.1), the difference is that a dc component is present in the unipolar waveform, and this results in a carrier component being present in the spectrum. For example, if the unipolar waveform consists of an alternating series ...1 0 1 0 1 0..., it appears just as the square wave of Fig. 2.7.2, and hence the modulated spectrum has side frequencies extending indefinitely on either side of the carrier, as shown in Fig. 12.9.4(a).

More generally, when the binary waveform is random, the baseband spectrum for the ac component is as shown in Fig. 3.4.3, and the modulated spectrum (in this case a spectrum density) is as shown in Fig. 12.9.4(b). Again, the dc component gives rise to a spike at the carrier frequency in the spectrum density plot. The presence of a component at the carrier frequency is important in the coherent detection of OOK waveforms, to be discussed shortly.

If  $B$  is the overall system bandwidth for the binary signal, the bandwidth for the modulated wave,  $B_T$  is

$$B_T = 2B \quad (12.9.3)$$

In the particular case where raised-cosine filtering is used on the baseband signal, then from Eq. (3.7.3), the modulated bandwidth becomes

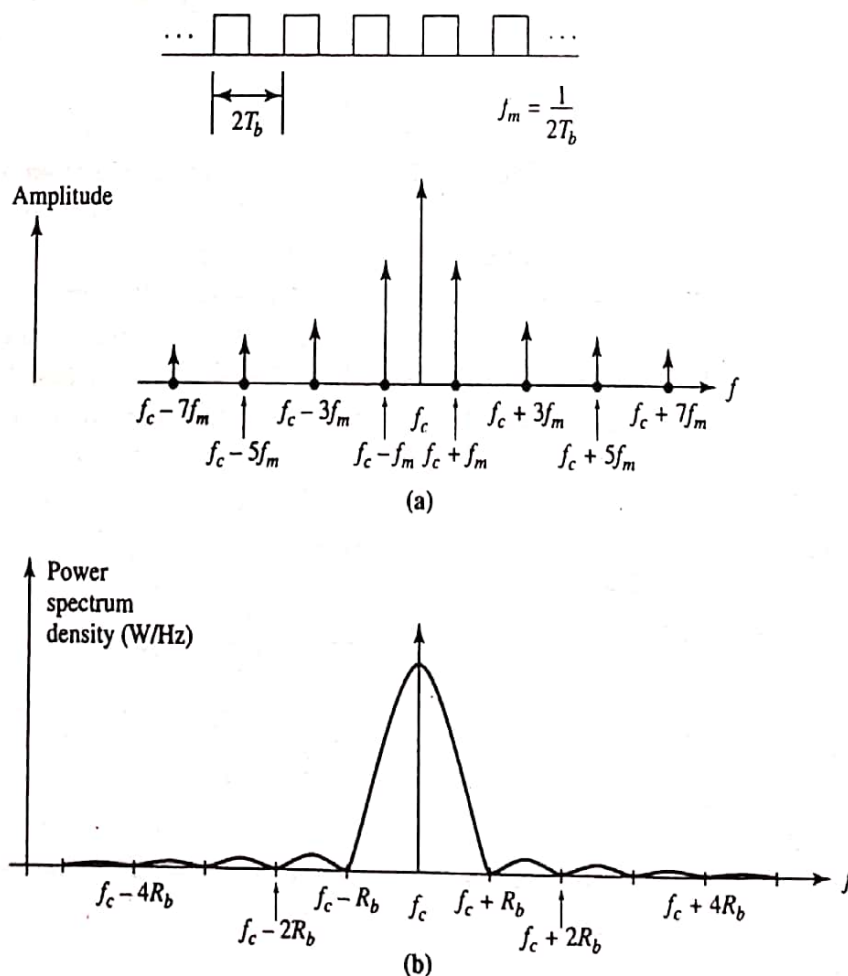


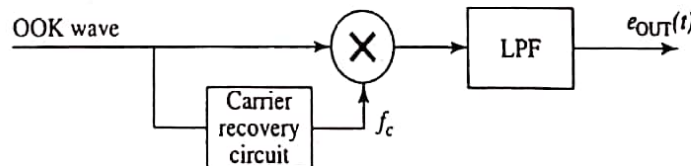
Figure 12.9.4 (a) Spectrum for OOK squarewave modulation. (b) More general picture of the OOK spectrum.

$$\begin{aligned}
 B_T &= 2 \frac{1 + \rho}{2T_b} \\
 &= (1 + \rho)R_b
 \end{aligned}
 \tag{12.9.4}$$

where  $R_b = 1/T_b$  is the transmitted bit rate. The ratio of bit rate to system bandwidth, the parameter defined by Eq. (3.7.11), becomes, for the OOK modulated wave

$$\begin{aligned}
 \alpha &= \frac{R_b}{B_T} \\
 &= \frac{1}{1 + \rho}
 \end{aligned}
 \tag{12.9.5}$$

Thus, for the ideal bandwidth  $\rho = 0$ , OOK provides a rate of 1 bps/Hz, and for  $\rho = 1$ , the rate is 0.5 bps/Hz. Demodulation of the OOK waveform may take place using a simple envelope detector as described in Section 8.11. A more efficient method is to use *synchronous* detection. This method is illustrated in block schematic form in Fig. 12.9.5.



**Figure 12.9.5** Synchronous demodulation of an OOK wave.

Synchronous detection requires a *carrier recovery* circuit, which is used to generate a local carrier component exactly synchronized to the transmitted carrier. As shown, the spectrum contains a component at the carrier frequency that can be used to phase lock the VCO in a PLL. Applying the locally generated carrier and the received signal to the multiplier results in an output

$$\begin{aligned}
 e_{\text{out}}(t) &= Ae(t) \cos(2\pi f_c t + \phi_c) \\
 &= Ae_m(t) [\cos(2\pi f_c t + \phi_c)]^2
 \end{aligned}
 \tag{12.9.6}$$

where  $A$  is an amplitude constant. Expanding the cosine squared term,

$$e_{\text{out}}(t) = Ae_m(t) \left[ \frac{1}{2} + \frac{1}{2} \cos(4\pi f_c t + 2\phi_c) \right]
 \tag{12.9.7}$$

The second harmonic carrier term is easily removed by filtering, leaving as the output

$$e_{\text{out}}(t) = \frac{A}{2} e_m(t)
 \tag{12.9.8}$$

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Thus the baseband signal is recovered. The constant term  $A/2$  is easily allowed for by adjustment of gain. The synchronous detection just described is also referred to as *coherent detection*. The demodulation of the OOK wave can also be carried out by using an envelope detector, as described in Section 8.11 (this also being known as noncoherent or nonsynchronous detection). Once the baseband signal is recovered, it can be used to regenerate new pulses, or the digital information can be recovered, as shown in Fig. 12.4.1(a).

The coherent detector is more complicated than the envelope detector, but it results in a lower probability of error for a given signal-to-noise input. The analysis will not be presented here, but the results are that the coherent OOK detection has a probability of error identical to that for the baseband system, which for optimum detection is given by Eq. (12.5.5) as

$$P_{be} = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{2N_o}} \quad (12.9.9)$$

The *optimum* noncoherent detector requires that  $E_b/N_o \gg 1$ , and for this condition

$$P_{be} \cong \frac{1}{2} e^{-E_b/2N_o} \quad (12.9.10)$$

#### EXAMPLE 12.9.1

Calculate the bit-error probability for OOK using (a) synchronous carrier demodulation and (b) nonsynchronous carrier demodulation when the bit energy to noise density ratio is 10 dB.

**SOLUTION** 10 dB = 10 : 1 energy ratio, and therefore:

$$(a) P_{be} = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{10}{2}} = 7.83 \times 10^{-4}$$

$$(b) P_{be} \cong \frac{1}{2} e^{-10/2} = 0.00337$$

In both cases, the bit energy is determined from the average carrier power. Let  $P_R$  represent the average received signal power; then for binary transmission

$$\begin{aligned} E_b &= P_R T_b \\ &= \frac{P_R}{R_b} \end{aligned} \quad (12.9.11)$$

A useful relationship can be derived between the  $E_b/N_o$  ratio and the S/N ratio. The noise power at the receiver input is  $P_N = N_o B_N$ , where  $B_N$  is the noise bandwidth. Hence



$$\begin{aligned} \frac{E_b}{N_c} &= \frac{P_R}{R_b} \cdot \frac{B_N}{P_N} \\ &= \frac{S}{N} \cdot \frac{B_N}{R_b} \end{aligned} \quad (12.9.12)$$

For the situation where  $B_N \cong B_T$ , where  $B_T$  is the system bandwidth, then using Eq. (3.7.11), which defines  $\alpha = R_b/B_T$ , gives

$$\alpha \frac{E_b}{N_o} = \frac{S}{N} \quad (12.9.13)$$

Thus the product of the two significant parameters for digital transmission,  $R_b/B_T$  and  $E_b/N_o$ , is equal to the received signal-to-noise ratio.

Where the transmission channel may be assumed distortionless and where raised-cosine filtering is used such that the receiver filter is the square root of the raised-cosine characteristic (matched by a similar filter at the transmitter, as is often the case with radio transmission), it can be shown that the noise bandwidth at the receiver is  $B_N = R_b$ . In this case, Eq. (12.9.13) becomes

$$\frac{E_b}{N_o} = \frac{S}{N} \quad (12.9.14)$$

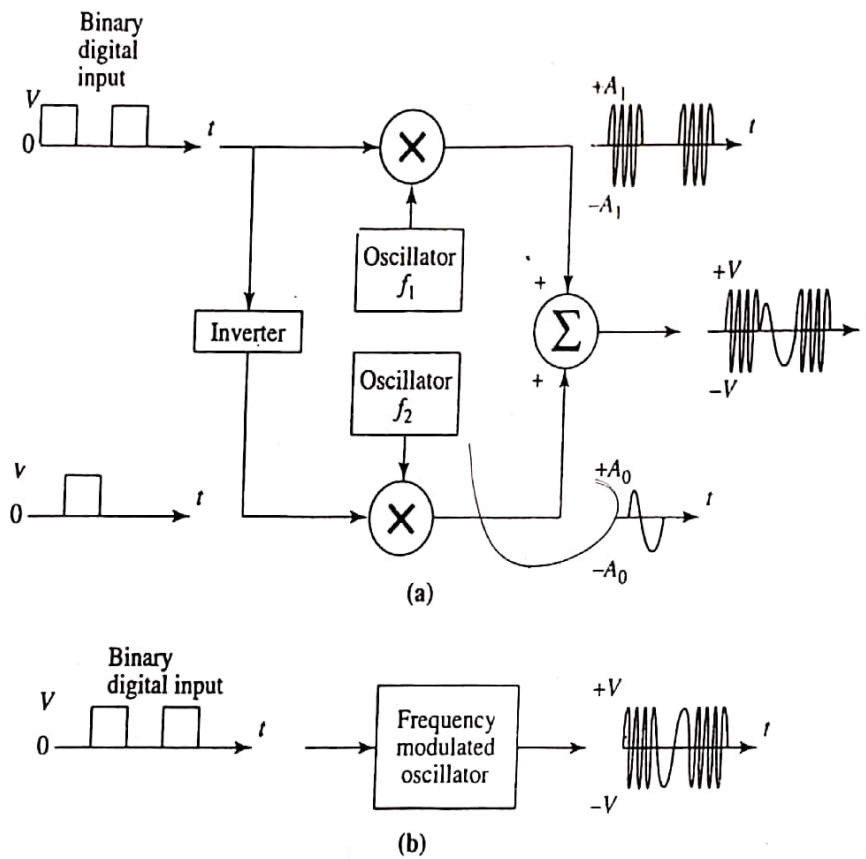
The usefulness of Eqs. (12.9.13) and (12.9.14) is that often the signal-to-noise ratio is the known quantity, while to calculate the bit-error probability, the  $E_b/N_o$  is the ratio required.

### Frequency Shift Keying

With frequency modulation, usually referred to as *frequency shift keying* (FSK), the carrier frequency is shifted in steps or levels corresponding to the levels of the digital modulating signal. In the case of a binary signal, two carrier frequencies are used, one corresponding to the binary 0 and the other to a binary 1. In general, for binary modulation the carriers can be represented by

$$\begin{aligned} \text{Binary 0: } e_0(t) &= A_0 \cos(2\pi f_0 t + \alpha_0) \\ \text{Binary 1: } e_1(t) &= A_1 \cos(2\pi f_1 t + \alpha_1) \end{aligned} \quad (12.9.15)$$

The two carriers may be generated from separate oscillators, independent of one another, and this is indicated by separate subscripts for the amplitudes and fixed phase angles. The combined signal can therefore have discontinuities in amplitude and phase, which are undesirable. Alternatively, the modulation can be achieved by frequency modulating a common carrier, which prevents the discontinuities from occurring. The block schematics for the two methods are shown in Fig. 12.9.6.



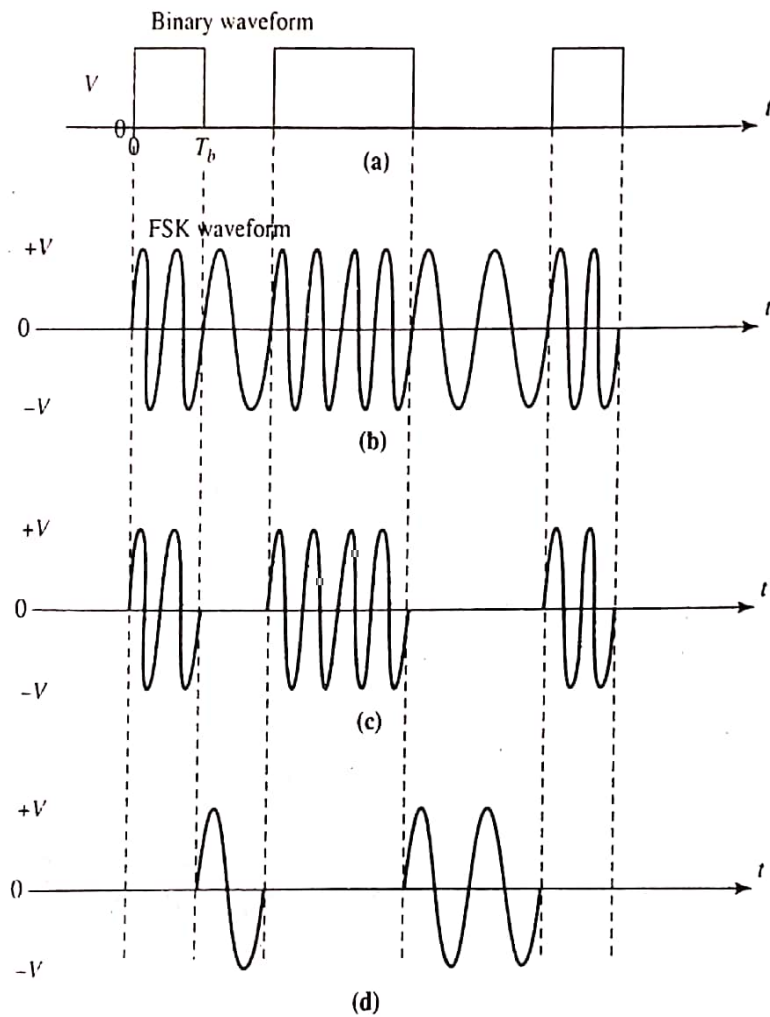
**Figure 12.9.6** (a) Separate oscillator method of realizing FSK. (b) Single oscillator method, or CPFSK.

Denoting the mean carrier frequency by  $f_c$ , then a binary 1 results in  $f_1 = f_c + \delta f$ , and a binary 0 in  $f_0 = f_c - \delta f$ , where  $2\delta f$  is the difference between the two signaling frequencies. The modulated signal is given by

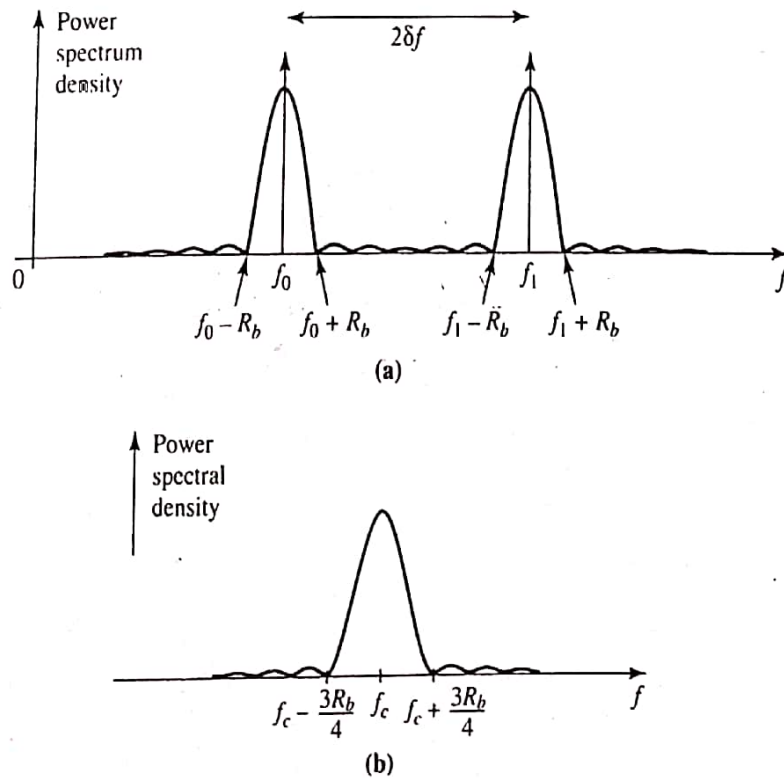
$$\begin{aligned} \text{Binary 0: } e_0(t) &= A \cos 2\pi f_0 t \\ \text{Binary 1: } e_1(t) &= A \cos 2\pi f_1 t \end{aligned} \quad (12.9.16)$$

where without loss of generality the fixed phase angle has been set equal to zero for each signal. Where a single oscillator is frequency modulated by the digital signal, the method is referred to as continuous phase frequency shift keying (CPFSK). The modulation could be represented more concisely by Eq. (10.2.3) for FM, but a detailed analysis of such a waveform is very complicated. A reasonable picture of the spectrum can be obtained by utilizing a different approach. Figure 12.9.7 shows the waveform for the CPFSK wave where, for convenience, an integer number of carrier cycles per bit period is shown.

By treating the CPFSK wave as two OOK waves as shown in the figure, the spectrum for the OOK wave shown in Fig. 12.9.4 can be used for each, and the resultant spectrum is as sketched in Fig. 12.9.8(a). A special and important case of CPFSK known as minimum shift keying (MSK) occurs when  $\delta f = R_b/4$ . This is the minimum separation for which correlation between the two signaling waveforms is zero. It can be shown that for any closer spacing the correlation between the waveforms results in an increased probability of bit error.



**Figure 12.9.7** (a) Binary waveform used in FSK. (b) FSK waveform. (c) and (d) Resolution of (c) into two OOK waves.



**Figure 12.9.8** Power spectra for (a) CPFSK with widely spaced signaling frequencies and (b) MSK.



For MSK, if it is assumed that the spectrum beyond the first nulls can be ignored, the overall spectrum bandwidth is seen to be

$$B_T = \frac{3}{2}R_b \quad (12.9.17)$$

This gives a bps/Hz figure of merit of  $\frac{2}{3}$ . In practice, it is found that the rate of decay of the spectrum outside a bandwidth given by  $B_T \cong R_b/2$  is very rapid, with the result that the bps/Hz figure can be improved to approximately unity.

Because FSK appears as two OOK waves, the coherent receiver can be constructed by using two separate OOK coherent detectors, as shown in Fig. 12.9.9(a). The outputs are combined to form a polar binary signal, which, for optimum detection, is then passed to a matched filter. Correlation between the two signaling frequencies results in general in an increased probability of bit error, but with MSK the correlation is zero and the expression for bit-error probability is the same as that for OOK, which is repeated here for convenience:

$$P_{be} = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{2N_0}} \quad (12.9.18)$$

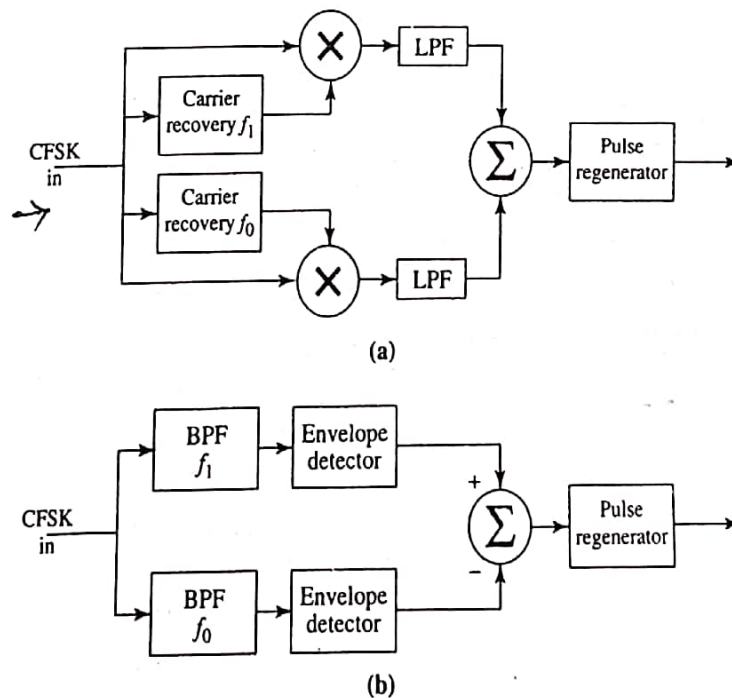


Figure 12.9.9 (a) Coherent and (b) noncoherent detection for CPFSK.

As pointed out in the introduction, when comparing OOK with CPFSK, it should be kept in mind that the bit energy with CPFSK is double that of OOK for the same carrier voltage levels at the receiver.

Noncoherent detection can also be used with FSK signals. Again, because FSK appears as two OOK waves, the noncoherent receiver need consist only of two separate paths with band-pass filters tuned to the individual frequencies,

as shown in Fig. 12.9.9(b). Each filter is followed by an envelope detector, as described in Section 8.11. The outputs are combined to form a polar waveform, which is then passed as input to the pulse regenerator operating at zero voltage threshold. When properly adjusted for optimum performance, the probability of bit error for the noncoherent FSK detector is given by

$$P_{be} \cong \frac{1}{2} e^{-E_b/2N_o} \quad (12.9.19)$$

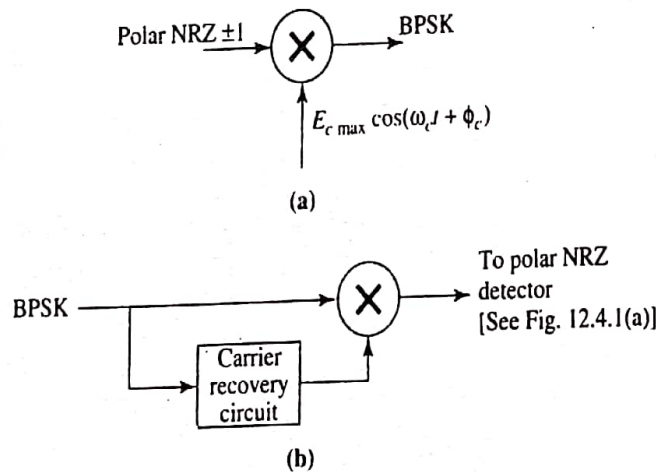
The noncoherent receiver is much simpler to build than the coherent receiver, and for many applications the degradation in bit-error probability is acceptable.

### Phase Shift Keying

With *phase modulation*, usually referred to as *phase shift keying* (PSK), the binary signal is used to switch the phase between  $0^\circ$  and  $180^\circ$ . It is also known as *phase reversal keying* (PRK). The modulated carrier is described by

$$e(t) = \begin{cases} E_c \max \cos(2\pi f_c t + \phi_c), & \text{binary 1} \\ E_c \max \cos(2\pi f_c t + \phi_c + 180^\circ), & \text{binary 0} \end{cases} \quad (12.9.20)$$

The circuit for implementing BPSK is a balanced modulator, as shown in Fig. 12.9.10(a).



**Figure 12.9.10** (a) Balanced modulator producing BPSK. (b) Detection of BPSK.

The modulating signal is polar NRZ, and when this has value +1, the modulated output is  $+1 \times E_c \max \cos(2\pi f_c t + \phi_c) = E_c \max \cos(2\pi f_c t + \phi_c)$ , and when it is -1, the modulated output is  $-1 \times E_c \max \cos(2\pi f_c t + \phi_c) = E_c \max \cos(2\pi f_c t + \phi_c + 180^\circ)$ . Coherent detection must be used with BPSK since the envelope does not contain the modulating information. The coherent BPSK receiver is shown in Fig. 12.9.10(b).

The BPSK modulator is similar to the OOK modulator, the difference being that no dc component is present in the modulating waveform and therefore no carrier component is transmitted. The spectrum is shown in Fig. 12.9.11. This is similar to that shown in Fig. 12.9.4, but with no spike at the carrier frequency.

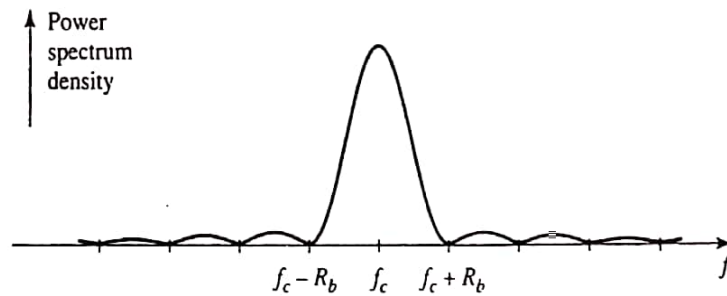


Figure 12.9.11 Spectrum for the BPSK wave.

The BPSK wave has in effect a DSBSC spectrum. With raised-cosine filtering on the baseband signal, the bps/Hz figure of merit is also given by Eq. (12.9.5) as  $1/(1 + \rho)$ . Coherent detection of BPSK followed by matched filter detection results in a bit-error probability given by

$$P_{be} = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_o}} \quad (12.9.21)$$

Just as binary baseband signals can be reformatted as  $M$ -ary signals with a consequent reduction in transmitted bandwidth, so  $M$ -ary level modulation can be used to similar effect. *Quadrature phase shift keying* (QPSK) utilizes four distinct levels of phase shift and is a widely used form of multi-level modulation. In this method a serial-to-parallel converter is used to convert the binary signal  $p(t)$  into two separate binary signals in which the bit period is doubled, as shown in Fig. 12.9.12(a) and (b). These two binary signals are labeled  $p_i(t)$  for *in-phase* and  $p_q(t)$  for *quadrature-phase* components, respectively. The in-phase component modulates a carrier to produce a BPSK signal, while the quadrature component modulates a carrier component shifted by  $90^\circ$  (hence the label *quadrature*), also to produce a BPSK signal. The two BPSK signals are added to produce the QPSK signal, the modulator states being as shown in Fig. 12.9.12(c). Thus the QPSK signal is equivalent to two BPSK signals, but with the carriers  $90^\circ$  out of phase with one another. Each BPSK waveform has a DSBSC spectrum, as described previously, and these spectra overlap. However, they do not interfere with one another because of the phase difference between the carriers. Thus QPSK signaling requires one-half the bandwidth of BPSK signaling for the same input bit rate in both cases, and the bps/Hz figure of merit is  $2/(1 + \rho)$ , where raised-cosine filtering is used.

Detection of QPSK is similar to that for BPSK with the difference that the recovered carriers must also have the  $90^\circ$  phase difference. A demodulator circuit is shown in Fig. 12.9.13. Assuming that the demodulated output is followed by a matched filter detector, the bit-error probability for QPSK is the same as that for BPSK as given by Eq. (12.9.21).