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⑩

Polarisation of Electromagnetic Waves :-

An electromagnetic wave such as light consists of a coupled oscillating electric field and magnetic field which are always perpendicular to each other; by convention, the "polarization" of electromagnetic waves refers to the direction of the electric field.

In longitudinal wave, the displacement from the equilibrium is along the direction of propagation. Sound waves, which are nothing but compression waves in air, are longitudinal.

Electromagnetic waves are transverse in nature. In a transverse wave displacement is perpendicular to the direction of propagation.

There are two dimensions perpendicular to any given line of propagation. Accordingly, transverse waves occur in two independent state of polarization.

- "Vertical" polarization $\tilde{f}_v(z,t) = \tilde{A} e^{i(kz-\omega t)} \hat{x}$,
- "Horizontal" polarization $\tilde{f}_h(z,t) = \tilde{A} e^{i(kz-\omega t)} \hat{y}$,

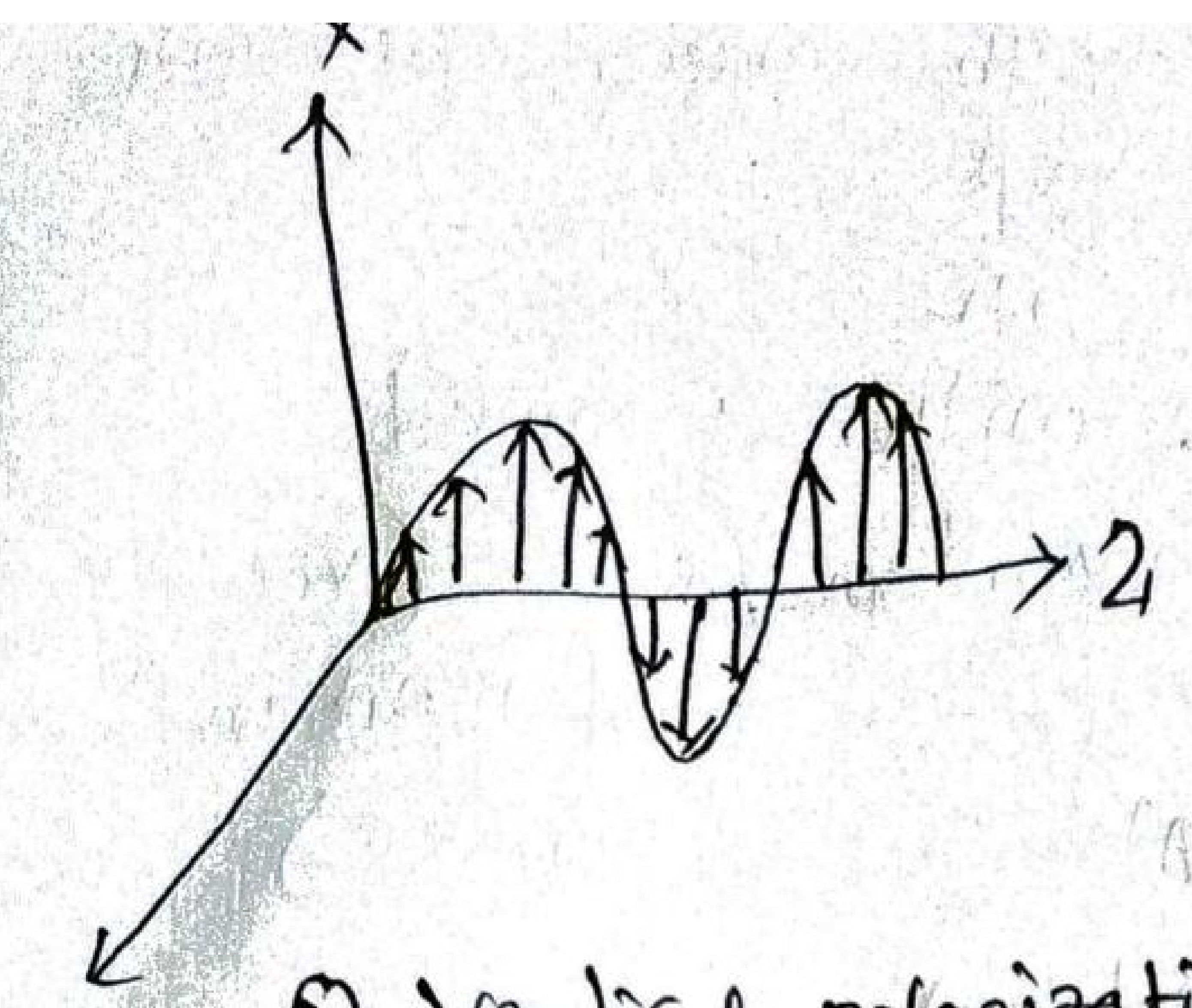
or along any other direction in the x-y plane $\tilde{f}(z,t) = \tilde{A} e^{i(kz-\omega t)} \hat{n}$.

The polarization vector $\hat{n}$ defines the plane of vibration. Because the waves are transverse $\hat{n}$ is perpendicular to the direction of propagation

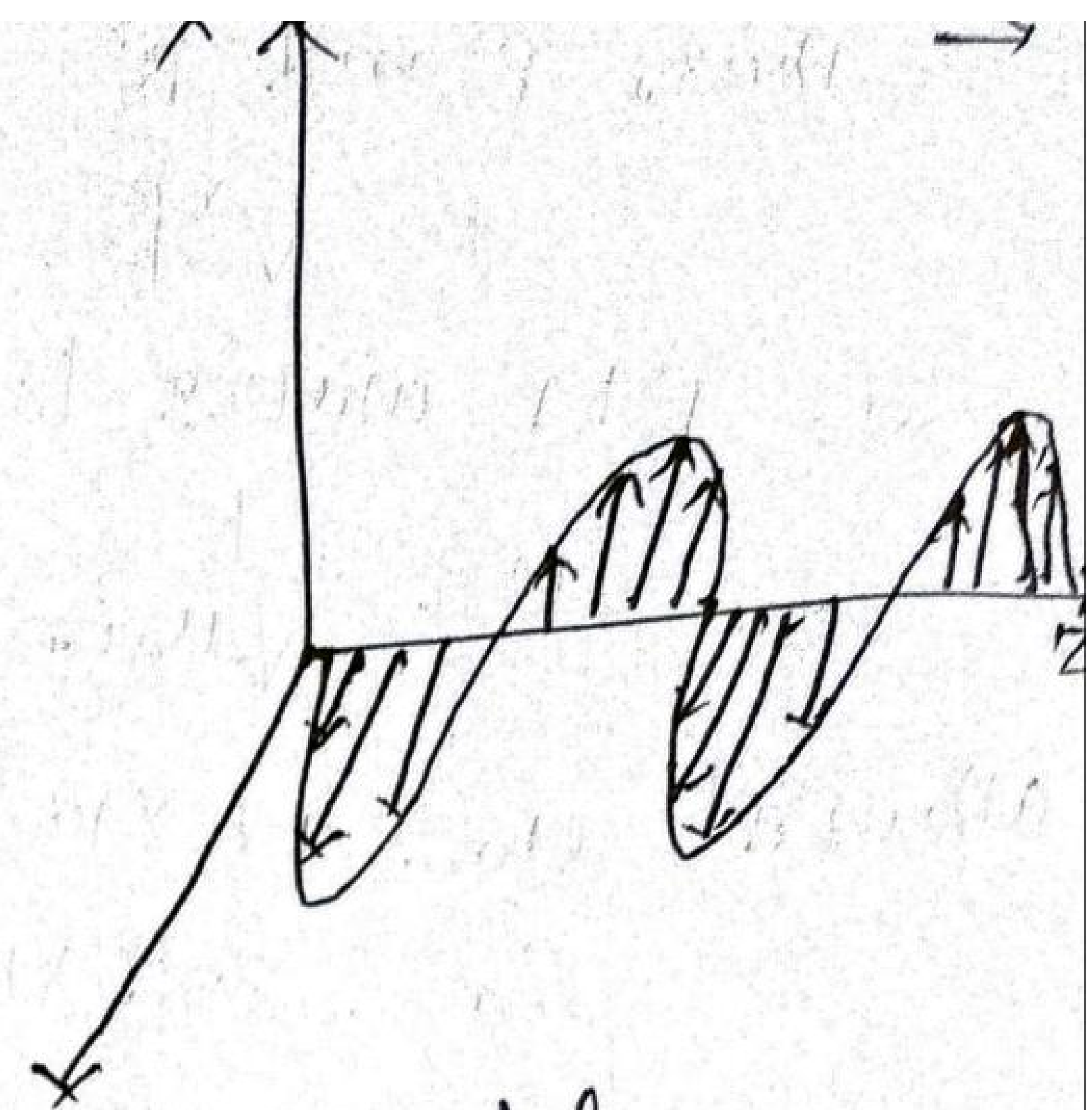
$$\hat{n} \cdot \hat{z} = 0$$

In terms of polarization angle θ,

$$\hat{n} = \cos\theta \, \hat{x} + \sin\theta \, \hat{y}$$



(a) Vertical polarization



(b) Horizontal. polarization

Electromagnetic wave in vacum :

• The wave equations for $\vec{E}$ and $\vec{B}$:

Maxwell's equations in free Space ($\rho = 0$ and $\vec{J} = 0$) Can be written as

(i) $\vec{\nabla} \cdot \vec{E} = 0$ (ii) $\vec{\nabla} \cdot \vec{B} = 0$

(iii) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (iv) $\vec{\nabla} \times \vec{B} = \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$

Taking Curl of equation (iii) and using equ^n (i) and (ii) we get.

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla} \times \left(-\frac{\partial \vec{B}}{\partial t}\right)$$

$$\Rightarrow \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B})$$

$$\Rightarrow -\vec{\nabla}^2 \vec{E} = -\frac{\partial}{\partial t}\left(\mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t}$$

Similarly $\nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$

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Thus, $\vec{E}$ and $\vec{B}$ Satisfy the wave equation

$$\nabla^2 f = \frac{1}{v} \frac{\partial^2 f}{\partial t^2}$$

So, EM waves travels with a speed

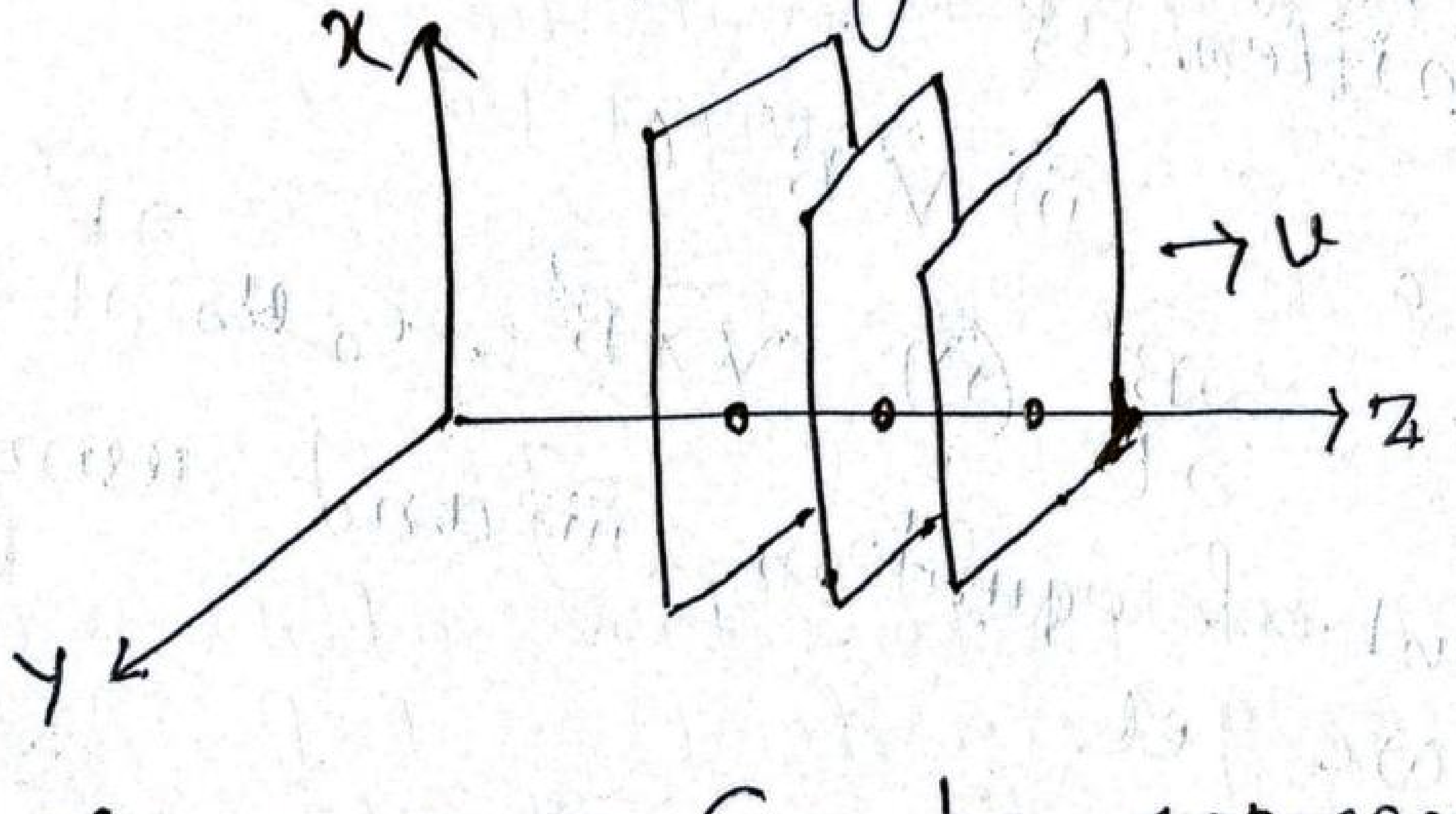
$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \, m/s = c \text{ (velocity of light in free Space)}$$

where, $\mu_0 = 4\pi \times 10^{-7} \, N/A^2$

$$\epsilon_0 = 8.86 \times 10^{-12} \, C^2/Nm^2$$

- Monochromatic plane wave :

Suppose waves are travelling in the z-direction and have no x or y dependence; these are called plane waves because the fields are uniform over every plane perpendicular to the direction of propagation.



The plane wave can be represent as

$$\tilde{E}(z,t) = \tilde{E}_0 e^{i(kz - \omega t)}, \quad \tilde{B}(z,t) = \tilde{B}_0 e^{i(kz - \omega t)}$$

where $\tilde{E}_0$ and $\tilde{B}_0$ are the (complex) amplitudes (the physical fields, of course are the real parts of $\tilde{E}$ and $\tilde{B}$)

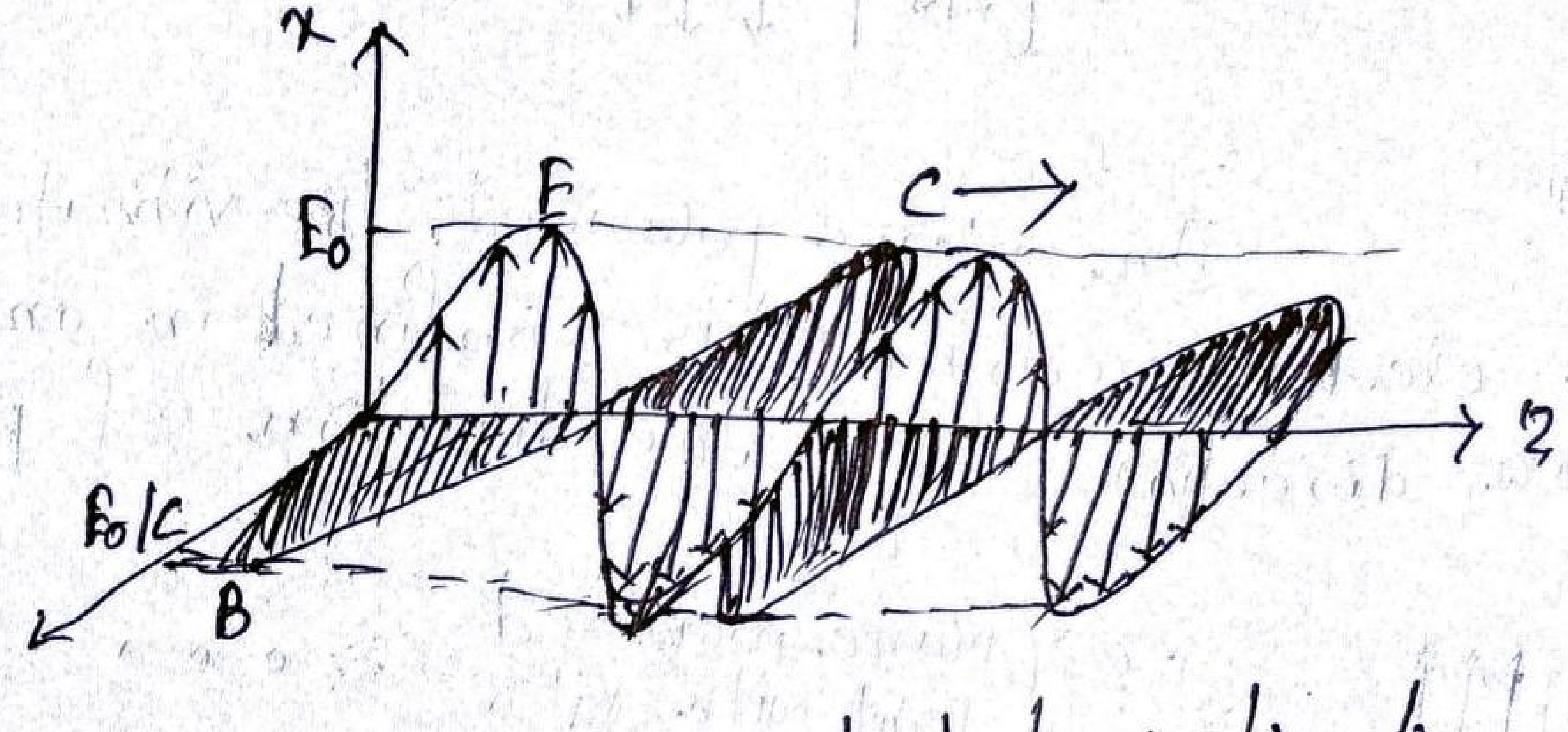
Since $\vec{\nabla} \cdot \vec{E} = 0$ and $\vec{\nabla} \cdot \vec{B} = 0$, it follows that $(\tilde{E}_0)_z = (\tilde{B}_0)_z = 0$, that is electromagnetic waves are transverse.

The electric and magnetic fields are perpendicular to the direction of propagation.

Also $\vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t} \Rightarrow -K(\tilde{E}_0)_y = \omega(\tilde{B}_0)_x \,;\, K(\tilde{E}_0)_x = \omega(\ldots$

In Compact form $\tilde{B}_0 = \frac{K}{\omega}(\hat{z}\times\tilde{E}_0)$

Evidently $\tilde{E}$ and $\vec{B}$ are in phase and mutually perpendicular, Their (real) amplitude are related by, $B_0 = \frac{K}{\omega}E_0 = \frac{1}{c}E_0$



There is nothing Special about the z direction, we can generalized the monochromatic plane waves traveling in an arbitary direction. The propagation vector or wave vector $\vec{k}$ points in the direction of propagation, whose magnitude is the wave number K. The Scalar product $\vec{K}\cdot\vec{r}$ is the appropriate generalization of Kz, So

$\tilde{E}(\vec{r},t) = \tilde{E}_0\, e^{i(\vec{K}\cdot\vec{r}-\omega t)}\,\hat{n}$

$\tilde{B}(\vec{r},t) = \frac{1}{c}\tilde{E}_0\, e^{i(\vec{K}\cdot\vec{r}-\omega t)}(\hat{K}\times\hat{n}) = \frac{1}{c}(\hat{K}\times\tilde{E})$

Where $\hat{n}$ is polarization vector.

Because $\vec{E}$ is transvrse, ($\vec{B}$ is also transverse) ! $\hat{n}\cdot\hat{K} = 0$

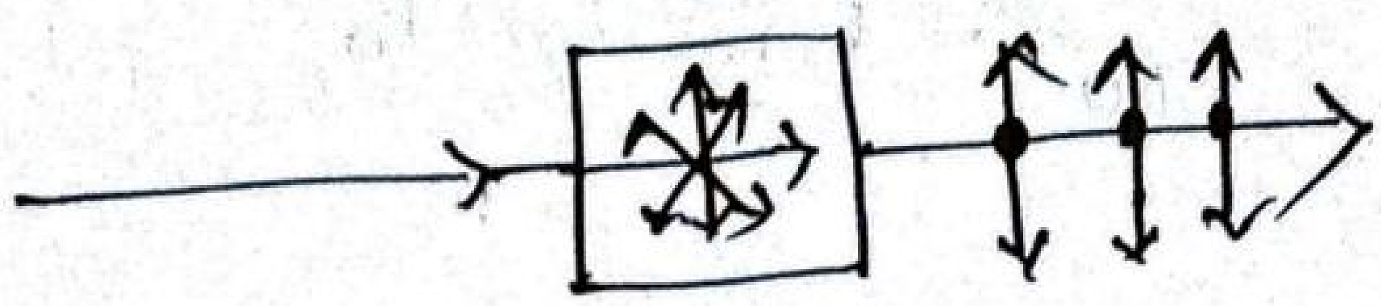
The actual (real) electric and magnetic fields in a monochromatic plane wave with propagation vector $\vec{k}$ and polarization $\hat{n}$ are

$\vec{E}(\vec{r},t) = E_0\cos(\vec{K}\cdot\vec{r}-\omega t+\delta)\hat{n}$

$\vec{B}(\vec{r},t) = \frac{1}{c}E_0\cos(\vec{K}\cdot\vec{r}-\omega t+\delta)(\hat{K}\times\hat{n})$

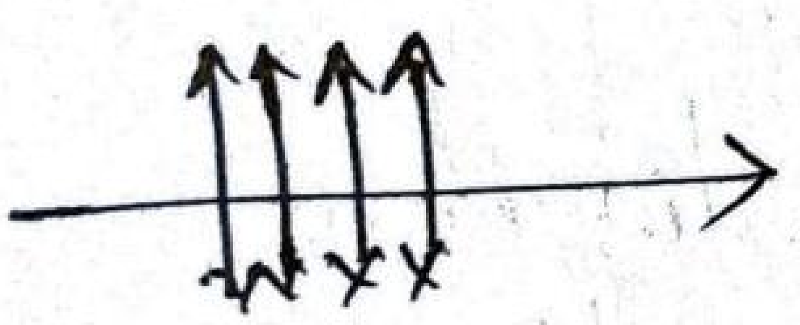
Define → (a) un polarised light
(b) plane polarised light

⇒ (a) In unpolarised light, the electric vector ($\vec{E}$) vibrate with equal amplitude in all possible direction $\perp^r$ to the direction of propagation



(unpolarised light)

(b) In plane polarised, the vibration of electric vector ($\vec{E}$) is confined in only one direction $\perp^r$ to the direction of propagation



→ (plane polarised light with its vibration in the plane of paper

(plane $\perp^r$ to the paper)

Define → (a) plane of vibration
(b) plane of polarisation

(a) The electric vector of a plane polarised light is confined to a particular plane, known as plane of vibration

This plane of vibration Contains the $\vec{E}$ - vector and propagtion vector ($\vec{K}$)

(b) The plane $\perp^r$ to the plane of vibration is known as plane of polarisation.

Define ⇒ (a) Circulary polarised light
(b) Left and right Circulary pol. right
(c) Elliptically polarised light.

⇒ VIII By the Superposition of two plane polarised waves under Suitable Condition, the resultant light vector rotate perpendicular ($\perp^r$) to the direction of propagation.

(a) If the magnitude of the resultant light vector remains Constant, then light vector appears to a Circle. Such a light vector is Said to be Circulary polarised.

(b) If resultant light vector appears to rotate clock wise then the light is Said to be Right Circularly polarised light.

If resitant light vector rotate anti-clockwise then light is said to be left Circularly polarised light.

(c) If the magnitude of the resultant light vector periodically between a maximum and a minimum value then light vectors appear to an elliptic path. Such a light is Said to be elliptically pol. light.

What do you mean by light vector

⇒ The electric Vector ($\vec{E}$) is Called light vector, because it is responsible for sensitive of vission.

prob

An unpolarised light of intensity 10 mw/cm^2 passed throught two Nicol with their principle Section at 30° to each other. Calculate the intensity of transmitted wave.

⇒

let, $I_1 \rightarrow$ intensity of polarised light produced by 1st Nicol.

Since, the incident unpolarised light can resolve into two orthogonal inchohert linear vibration each of intensity $I_0/2$ and only one component parallel to the transmission axis will pass through the 1st Nicol.

Thus $I_1 = I_0/2 = 5$ mw/cm^2

According to Malu's law the intensity of light transmitted through 2nd Nicol is

$$I_2 = I_1 \cos^2\theta$$

$$= 5 \times \cos^2 30 \text{ mw/cm}^2$$

$$= 5 \times \frac{3}{4} = 3.75 \text{ mw/cm}^2$$

Double refraction :- When an ordinary ray of unpolarised light is made incident on a crystal like calcite, quartz etc., it is splited into two rays. One of those ray obey the law of refraction, called o-ray. While other does not obey those law, called E-ray. This phenomenon of splitting of ray into two (during its passage through the crystal) is known as Double refraction.

The crystal which shows the double refraction is called Double refracting crystal.

uniaxial crystal :- A double refracting crystal which have only one optic axis is called a uniaxial crystal. Ex- Calcite, quartz.

principle Section :- principle Section of a crystal is its Section by a plane which passes through the optic axis of the crystal and is $\perp^r$ to its two opposite refracting faces.

positive Crystal :- If, the velocity of o-ray is greater than the E-ray or if the r.i of E-ray is less than the r.i of o-ray in the direction $\perp^r$ to o.a, then the crystal is said to be positive crystal.

$$\boxed{\begin{array}{c} \mu_o < \mu_e \\ \hline v_e < v_o \end{array}}$$ for '+ve' crystal

$\left\{ \begin{array}{l} \text{positive} \\ \mu_o < \mu_e \\ \mu = c/v \end{array} \right.$

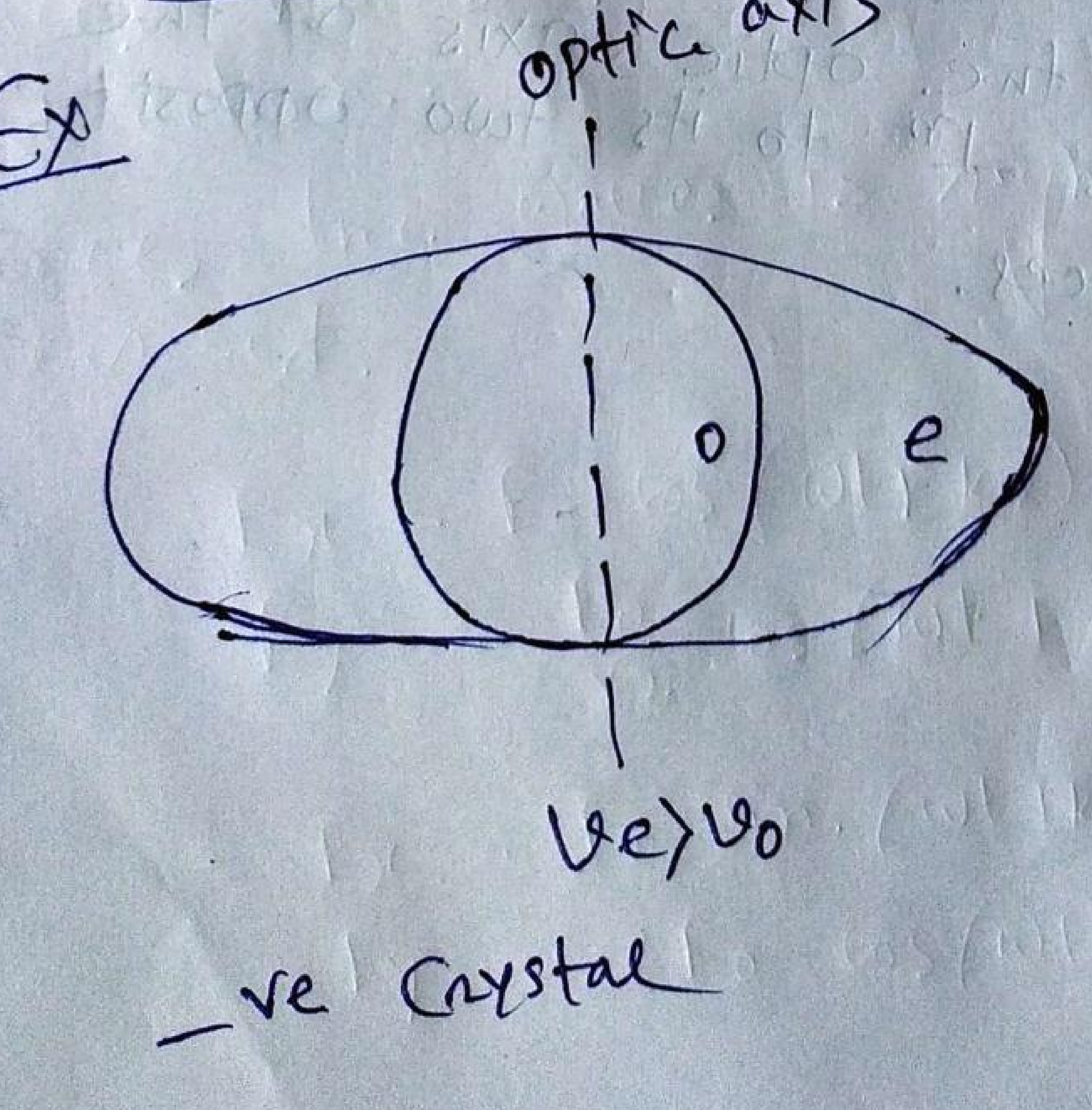
Ex - Quartz

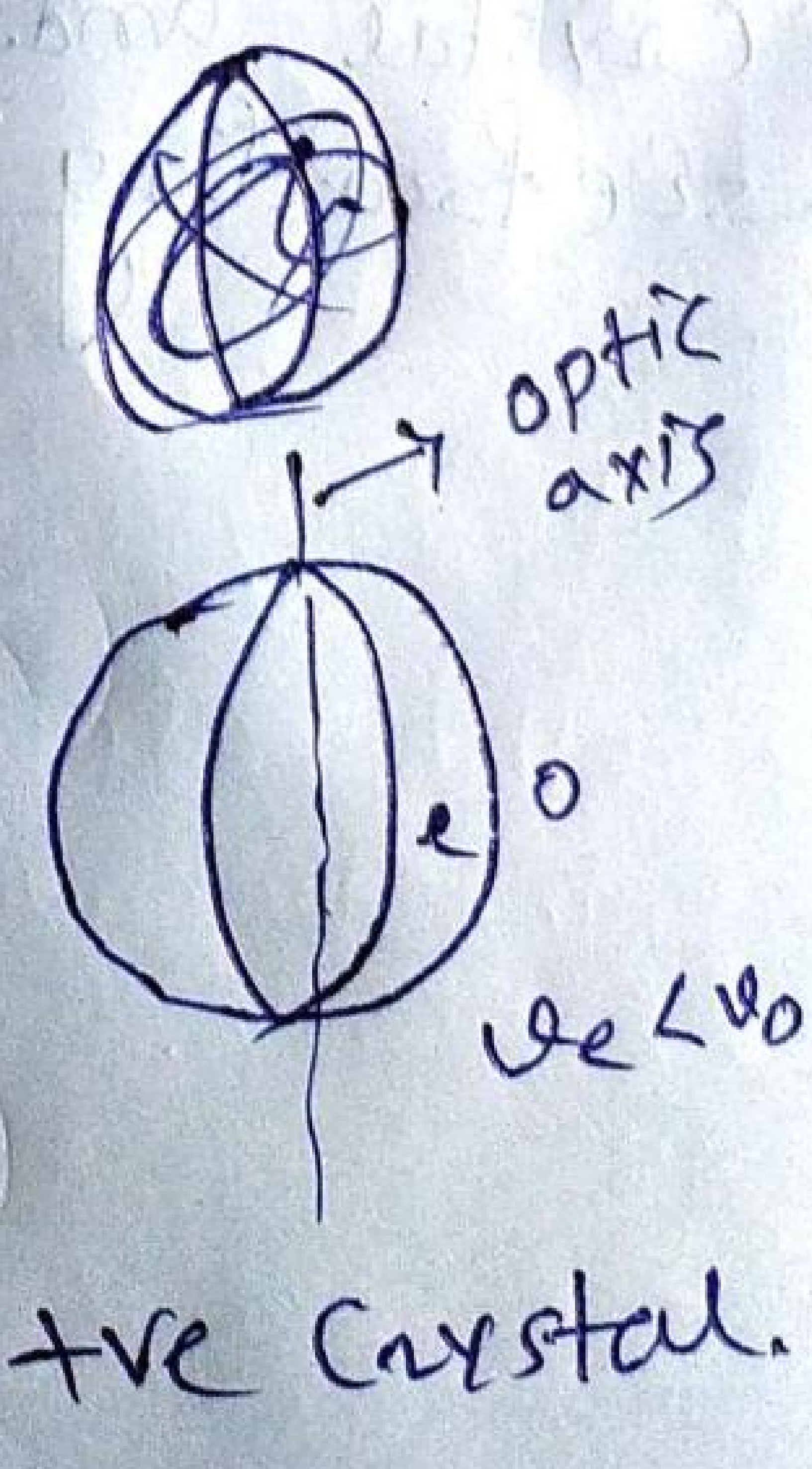
Negative Crystal :- If the velocity of o-ray is less than E-ray or if r.i of o-ray is greater than E-ray in the direction $\perp^r$ to o.a then the crystal is said to be Negative Crystal,

[o.a → optic axis]

$$\boxed{\begin{array}{c} \mu_o > \mu_e \\ \hline v_e > v_o \end{array}}$$ for '-ve' Crystal

Ex





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write down the electric and magnetic field for a plane monochromatic wave of amplitude E_0, frequency ω and phase angle zero that is
(a) traveling in the y-direction and polarized in the x-direction.
(b) traveling in the direction from the origin to the point (1,1,1) with polarization parallel to XY plane.

$\Rightarrow$ (a) $\vec{E} = E_0 \cos(ky - \omega t)\hat{x}$

$$\vec{B} = \frac{1}{c}\hat{k}\times\vec{E} = \frac{E_0}{c}\cos(ky - \omega t)(\hat{y}\times\hat{x})$$

$$= -\frac{E_0}{c}\cos(ky-\omega t)\hat{z}$$

(b) $\vec{k} = \frac{\omega}{c}\left(\frac{\hat{x}+\hat{y}+\hat{z}}{\sqrt{3}}\right)\hat{n} = \alpha\hat{x}+\beta\hat{y}$

$\Rightarrow \hat{n}\hat{k} = 0$

$\Rightarrow \alpha = -\beta = \frac{1}{\sqrt{2}} \Rightarrow \hat{n} = \frac{\hat{x}-\hat{y}}{\sqrt{2}}$

$$\vec{E}(r,t) = E_0 \cos(\vec{k}.\vec{r} - \omega t)\hat{n}$$

where $\vec{k}.\hat{r} = \frac{\omega}{\sqrt{3}c}(x+y+z)$

$$\vec{E}(r,t) = E_0\cos\left[\frac{\omega}{\sqrt{3}c}(x+y+z) - \omega t\right]\left(\frac{\hat{x}-\hat{y}}{2}\right)$$

$$\vec{B} = \frac{1}{\omega}\vec{k}\times\vec{E}$$

$$= \frac{1}{\omega}E_0\cos\left[\frac{\omega}{\sqrt{3}c}(x+y+z) - \omega t\right]\left[\frac{\omega}{c}\left(\frac{\hat{x}+\hat{y}+\hat{z}}{\sqrt{3}}\right)\times\left(\frac{\hat{x}-\hat{y}}{2}\right)\right]$$

$$\vec{B} = \frac{E_0}{2\sqrt{3}c}\cos\left[\frac{\omega}{\sqrt{3}c}(x+y+z) - \omega t\right](\hat{x}+\hat{y}-2\hat{z})$$

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Q Find the state of polarization for the following electric field components →

$$E_x = E_0 \sin(\omega t + kz)$$
$$E_y = E_0 \cos(\omega t + kz)$$

⇒ Since, amplitude are same (E_0 say)

Thus we can write, $E_x^2 + E_y^2 = E_0^2$

This is the eqn of a circle. Hence this light is circularly polarised light.

For X-Y plane ⇒ $z = 0$

Thus, $E_x = E_0 \sin\omega t$

$E_y = E_0 \cos\omega t$ $\qquad \omega t = 2\pi$

Now, we can calculate E_x and E_y for different time, i.e

(i) at, $t = 0$; $E_x = 0$; $E_y = E_0$

(ii) at, $t = T$; $E_x = E_0$; $E_y = 0$

(iii) at $t = T/2$; $E_x = 0$; $E_y = -E_0$

Hence, vibration rotation clock wise direction.

Hence, the waves is right circularly polarised.

H.W
$$E_x = E_0 \cos(\omega t + kz)$$
$$E_y = \frac{E_0}{\sqrt{2}} \cos(\omega t + kz + \pi)$$

H.W
$$E_x = E_0 \sin(\omega t + kz)$$
$$E_y = 2E_0 \cos(\omega t + kz)$$

H.W

$$E_x = E_0 \sin(\omega t + kz)$$
$$E_y = E_0 \cos(\omega t + kz + \pi)$$

H.W An e.m is expressed as —

$E_x = A\cos(\omega t+\theta)$; $E_y = B\cos(\omega t+\theta)$. for what magnitude of A, B, θ, ϕ will the wave be (i) plane polarised (ii) Circularly polarised

$\Rightarrow$ $E_x = A\cos(\omega t+\theta)$
$E_y = B\cos(\omega t+\phi)$

(i) for plane pola. light the amplitudes are same and $\theta = \phi$

i,e $A = B$, $\theta = \phi$ are required. Condition

So that $\boxed{E_x = E_y}$ i,e plane pol. light.

(ii) for Circularly polarised light amplitudes are same but $\phi = \theta + \pi/2$

then $E_y = A\cos(\omega t + \theta + \pi/2)$

$E_y = A\sin(\omega t+\theta)$

$E_x = A\cos(\omega t+\theta)$

$E_x^2 + E_y^2 = A^2 \Rightarrow$ circularly polarised

Thus $\boxed{A = B}$ and

$$\boxed{\phi = \theta + \pi/2}$$

Required condition

4.15 FRESNEL DIFFRACTION

The diffraction we have already discussed are of two types, namely Fraunhoffer diffraction and Fresnel diffraction. In the previous topic, we have dealt with the Fraunhoffer diffraction. In this part, we will deal with the Fresnel Diffraction. The assumptions of Fresnel's diffraction are as follows :

(i) Each element of a wavefront emits secondary wave continuously.

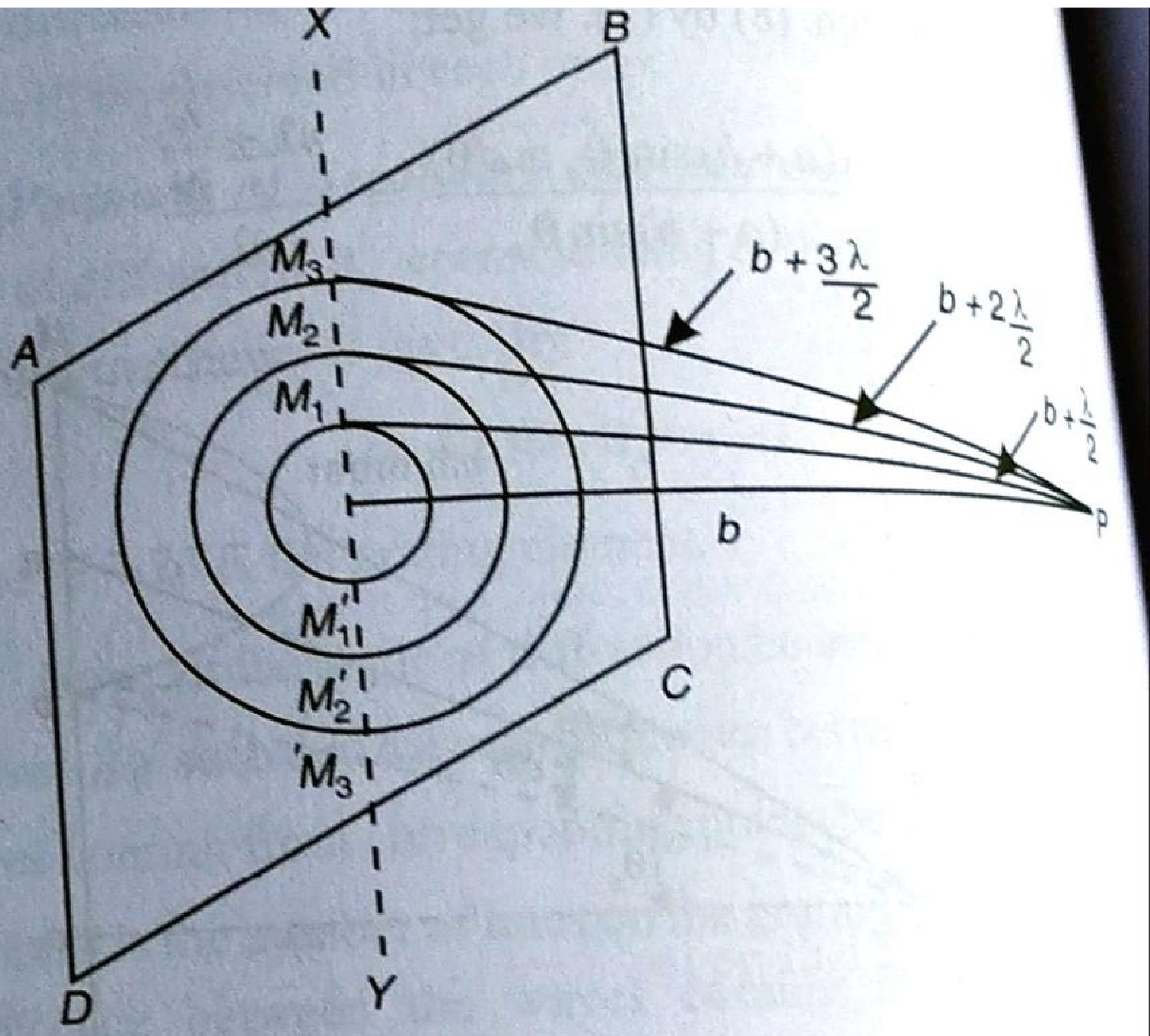
(ii) The effect of any element on the wavefront is maximum along a direction normal to it and decreases as the angle of inclination increases in either direction.

(iii) The amplitude and hence the intensity at any point is determined by combining the effects of the waves reaching there.

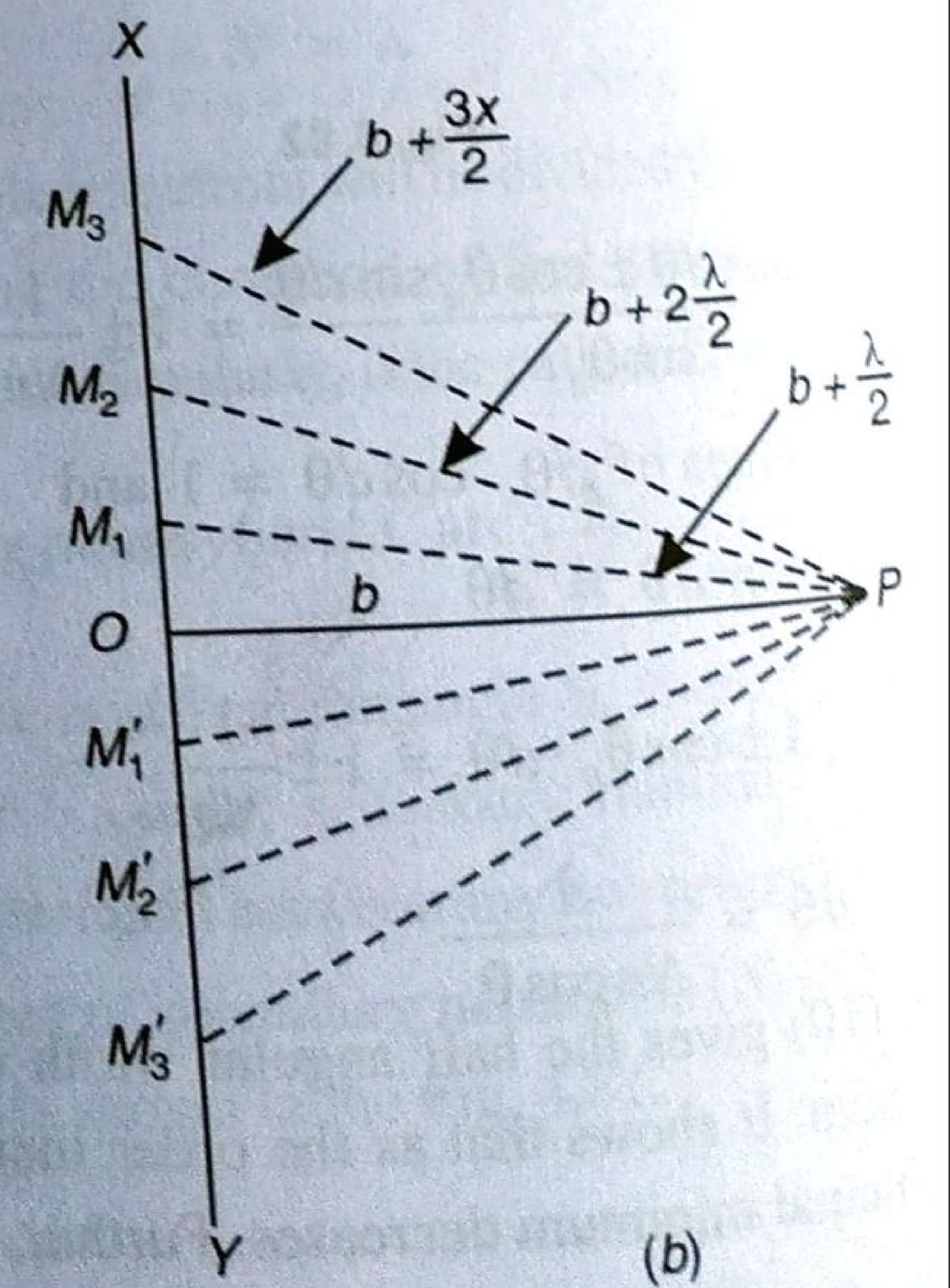
1. Fresnel's Half period zones

Consider a plane wavefront $ABCD$ perpendicular to plane of paper as shown in Fig. 4.83. Let P be the point of observation and OP is such that $OP \perp XY$. Let $OP = b$.

According to Huygen's principle, every point on the wavefront $ABCD$ becomes a source of secondary waves. The direction OP satisfies the assumption (2). In otherwords, the secondary waves emanating from O travel the shortest distance. As we move away from O, the distance of the secondary sources from P goes on increasing, the angles OPM_1 also increases, thus the contribution (effect) at P also decreases.



(a)



(b)

Fig. 4.83

For simplicity, Fresnel divided the wavefront into annular regions or zones of increasing radii such that the mean distance of a zone from the point of observation P increases in steps of $\lambda/2$, as one moves outwards, where λ is the wavelength of light. The secondary waves coming from various zones change their phase by π from one zone to the next. Fresnel called these zones as the *half period zones* or *half period elements*. To obtain these half period zones, with P as center and radii equal to b, $b+\frac{\lambda}{2}$, $b+\lambda$, $b+\frac{3\lambda}{2}$ ... draw concentric spheres as shown in the Fig. 4.84. The area enclosed by the first circle is called the *first half period zone* or *first half period element*, the area enclosed between the second and the first circle is called the *second half period zone* and so on.

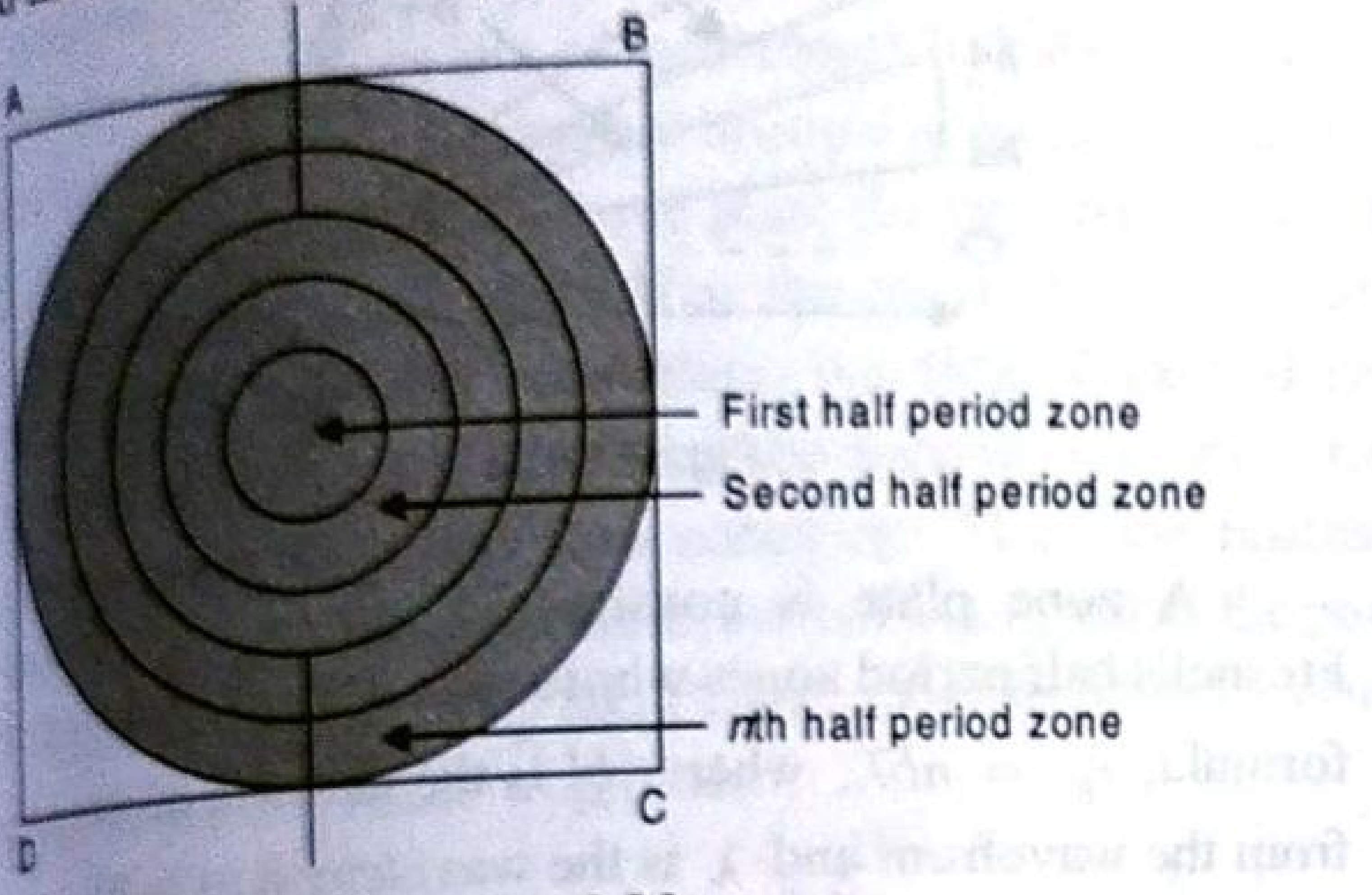


Fig. 4.84

The mean distance of the first half period zone from P is,

$$\frac{b+b+\frac{\lambda}{2}}{2} = b+\frac{\lambda}{4}, \quad \ldots(1)$$

and that of the second half period zone is $b+\frac{3\lambda}{4}$, of third is $b+\frac{5\lambda}{4}$ etc. ...(2)

Let the displacement at P due to wave coming from the first half period zone be,

$$y_1 = a_1 \sin\frac{2\pi}{\lambda}(vt-x) = a_1\sin(\omega t-\phi) \quad \ldots(3)$$

It can be easily seen that the displacement at P due to second half period zone is

$$y_2 = -a_2\sin(\omega t-\phi); \text{ and so on.}$$

$\therefore$ The resultant displacement at P due to all half period zone is :

$$\begin{aligned} y &= y_1+y_2+y_3+\cdots \\ &= (a_1-a_2+a_3-a_4+\cdots)\sin(\omega t-\phi) \\ &= A\sin(\omega t-\phi) \end{aligned} \quad \ldots(4)$$

or, the resultant amplitude at P is

$$A = a_1 - a_2 + a_3 - a_4 + \cdots \quad \ldots(5)$$

The amplitudes a_1, a_2, a_3, a_4 ... etc. depends upon (*i*) the area of the corresponding zones, (*ii*) the mean distance between various zones and the point P and (*iii*) the obliquity of rays reaching P from corresponding zones.

(*i*) *Area :* The area of the nth half period zone is obtained by subtracting the area enclosed by the circle of radius r_{n-1} from the circle of radius r_n.

$$\therefore \text{ Area of } n\text{th zone} = \pi(r_n^2 - r_{n-1}^2)$$

$$= \pi\left[\left(b+\frac{n\lambda}{2}\right)^2 - b^2\right] - \pi\left[\left(b+\frac{(n-1)\lambda}{2}\right)^2 - b^2\right]$$

$$= \pi b\lambda \quad (\because \text{ neglecting terms in } \lambda^2, \text{ being very small}) \ldots(6)$$

As area of nth period zone is independent of n, under approximation the contribution of each half period zone is very nearly same.

(*ii*) *Distance :* The amplitude of a wave reaching P depends upon the distances of the half period zone and the point P. As the distance of the half period zone increases from P with its order, in steps of $\lambda/2$, the contribution to the resultant amplitude decreases.

(*iii*) In accordance with Fresnel's assumption, since the inclination of second, third, fourth, ... half period zones (or h, p, e's) goes on increasing, the effect of these elements on intensity of the point P goes on decreasing.

Now, keeping in mind, these three factors, we conclude that the amplitudes $a_1, a_2, a_3, \ldots$ decreases regularly and may be considered to form an arithmetic progression. Thus, we have :

$$\begin{aligned} A &= a_1 - a_2 + a_3 - a_4 + \ldots + a_n \\ &= a_1 - \left(\frac{a_1+a_3}{2}\right) + a_3 - \left(\frac{a_3+a_5}{2}\right) + a_5 \cdots + a_n \\ &= \frac{a_1}{2} + \frac{a_n}{2} \\ &= \frac{a_1}{2}, \text{ as } \frac{a_n}{2} \text{ is negligibly small.} \end{aligned} \quad \ldots(7)$$

Hence, the resultant amplitude at P due to whole of the wavefront is just equal to half of the amplitude due to the first half period zone.

If we place an obstacle at O, of radius less than $\sqrt{b\lambda}$, the light reaching P will be completely cut off (such that the amplitude at P due to the first half period zone is only $\frac{a_1}{2}$).

4.16 POLARIZATION OF LIGHT

Light is an electromagnetic wave in which electric and magnetic field vectors vary sinusoidally, perpendicular to each other as well as perpendicular to the direction of propagation of light wave.

In common light sources, *e.g.*, sun or a bulb, atoms are the elementary radiators of light. As these atoms act independently, light propagated from such sources in a given direction consists of many independent waves whose planes of vibration are randomly oriented about the direction of propagation as shown in the Fig. 4.87(a). Such light waves are said to be ***unpolarized*** **or** ***polarized randomly.***

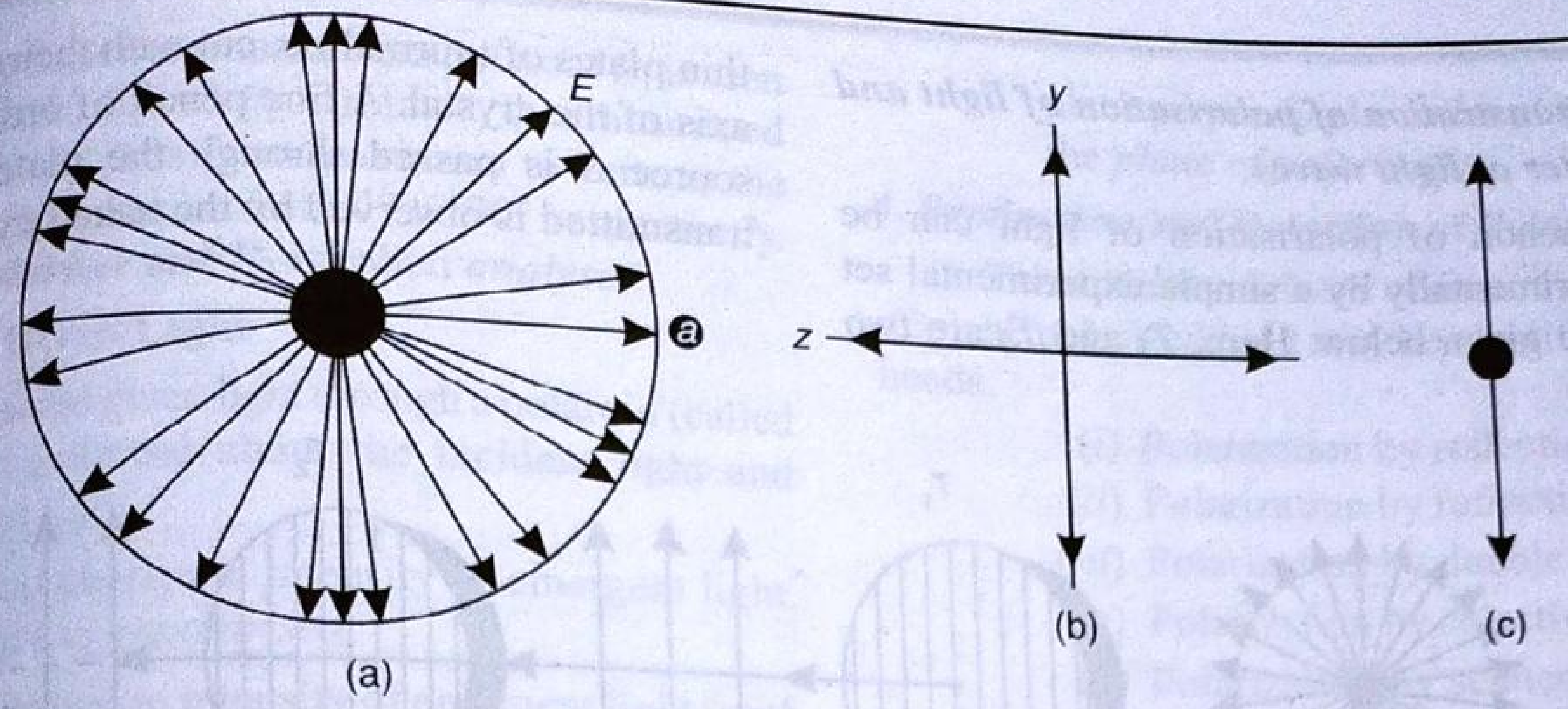


Fig. 4.87

Polarization is the property of electromagnetic waves, such as light, that describes the direction of the electric field. The polarization of a transverse wave describes the direction of oscillation in the plane perpendicular to the direction of travel. The term polarization distinguishes between the different orientations of oscillatory waves of the same path.

Longitudinal waves such as sound waves do not exhibit polarization, because for these waves the direction of oscillation is along the direction of travel.

All electromagnetic waves propagating in free space or in a uniform materials of infinite extent have electric and magnetic fields perpendicular to the direction of propagation. Ignoring the magnetic field vector (as it is ⊥ to E and proportional to it), $\vec{E}$ can be divided into two perpendicular components z and y.

In principle, each electric field of Fig. 4.87(a) can be resolved into y and z components. We can then find the net electric field along the y-axis and along the z-axis separately as shown in the Fig. 4.87(b). Thus, unpolarized light can be thought of as the superposition of two polarized waves whose planes of vibration are perpendicular to each other. The symbol of unpolarized light is shown in Fig. 4.87(c).

When unpolarized light is passed through a tourmaline crystal cut with its face parallel to its crystallographic axis AB, Fig. 4.88, only those vibrations of light pass through the crystal, which are parallel to AB. All other vibrations are absorbed. That is why intensity of light emerging from the crystal is reduced.

The emergent light from the crystal is said to be plane polarized light.

"*The phenomenon of restricting the vibrations of light (electric vector) in a particular direction, perpendicular to the direction of wave motion is called polarisation of light.*"

The tourmaline crystal acts as a *polariser.* Thus, electromagnetic waves are said to be polarized when their electric field vectors are all in a single plane, called the *plane of oscillation/vibration.* Light waves from common sources are *unpolarized* or *randomly polarized.*

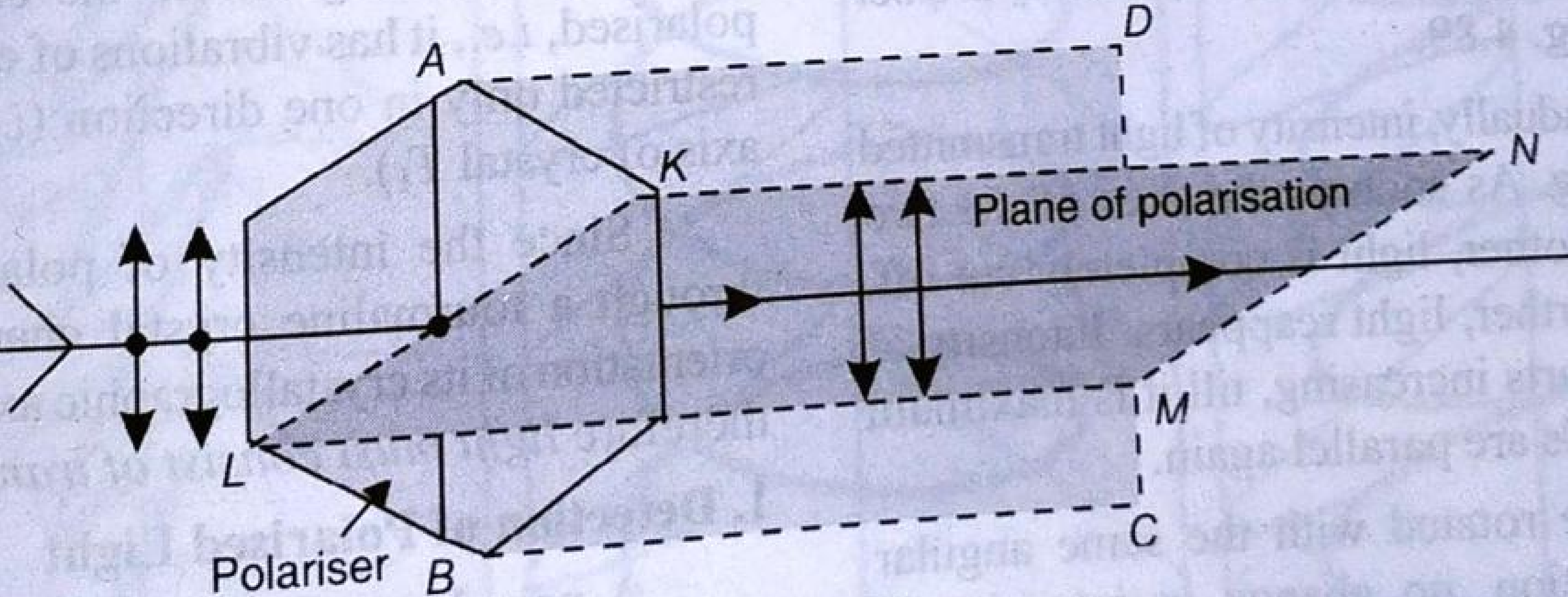


Fig. 4.88

The plane $ABCD$ in which the vibrations of polarized light are confined is called the *plane of vibration.* The plane $KLMN$ which is perpendicular to the plane of vibration is defined as the *plane of polarization.*

> **Note :** When originally unpolarized light is passed through a polarizing sheet, the transmitted *intensity is half the original intensity.* In actual practice, the transmitted intensity may be even somewhat less than half the original intensity.

Experimental demonstration of polarisation of light and transverse character of light waves

The phenomenon of polarisation of light can be demonstrated experimentally by a simple experimental set up, shown in figure given below. Here, T_1 and T_2 are two thin plates of tourmaline, cut with their faces parallel to the axis of the crystal. A fine pencil of ordinary light from the source S is passed through the plate T_1; and the light transmitted is observed by the naked eye.

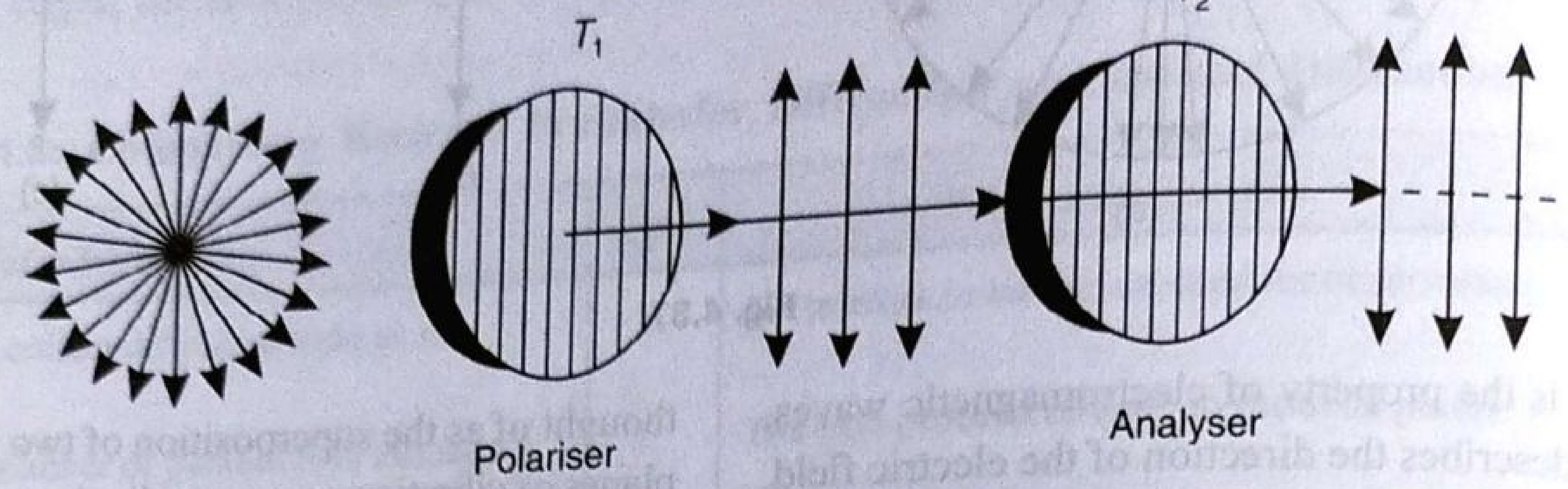


Fig. 4.88

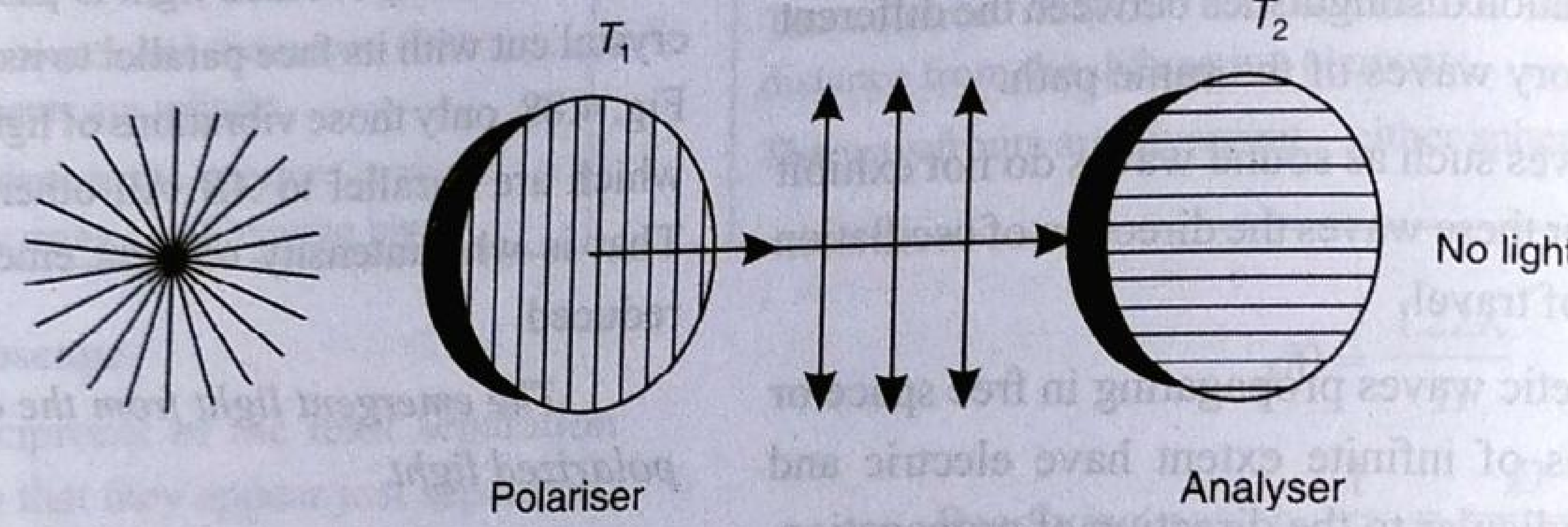


Fig. 4.89

When the plate T_1 is rotated about the direction of propagation of light as axis, the intensity and character of transmitted light remains the same (except for slight change in colour due to selective absorption within the crystal). Let the second plate T_2 be placed in the path of light transmitted from T_1. We observe that *intensity and character of light transmitted by T_1 and T_2 remain unaffected only when T_1 and T_2 are set with their axes parallel,* Fig. 4.89.

When T_2 is rotated gradually, intensity of light transmitted from T_2 goes on decreasing. As soon as the axes of the two crystals are at 90° to each other, light is completely cut off, Fig. 4.89. On rotating T_2 further, light reappears. Intensity of light transmitted from T_2 starts increasing, till it is maximum when axes of the two crystals are parallel again.

If both T_1 and T_2 are rotated with the same angular velocity in the same direction, no change in intensity of transmitted light is observed.

The phenomenon can be explained only when we assume that *light waves are transverse.* Now, the light falling on T_1 has transverse vibrations of electric vector lying in all possible directions. The crystal T_1 allows only those vibrations to pass through it, which are parallel to its axis. When the crystal T_2 is introduced with its axis kept parallel to the axis of T_1, the vibrations of electric vector transmitted by T_1 are also transmitted through T_2.

However, when axis of T_2 is perpendicular to axis of T_1, vibrations of electric vector transmitted from T_1 are normal to the axis of T_2. Therefore, T_2 does not allow them to pass and hence eye receives no light.

Light coming out of the crystal T_1 is said to be polarised, *i.e.*, it has vibrations of electric vector which are restricted only in one direction (*i.e.*, parallel to the optic axis of crystal T_1).

Since the intensity of polarized light on passing through a tourmaline crystal changes, with the relative orientation of its crystallographic axes with that of polariser, therefore *light must consist of transverse waves.*

1. Detection of Polarised Light

A naked eye cannot distinguish between polarised and unpolarised light. A crystal can be used for making this distinction. A calcite crystal or quartz crystal, or a nicol prism (made from calcite crystal) can be used as *polariser* as well as *analyser* of polarised light.

When unpolarised light is seen through a single crystal (polaroid), intensity of transmitted light decreases, on account of polarisation. On rotating the crystal, intensity of polarised light does not change.

However, when light transmitted from polaroid P_1 is seen through another polaroid P_2 and P_2 is rotated, the transmitted fraction of light from P_2 falls from maximum to zero as the angle between P_1 and P_2 varies from 0° to 90° respectively. Here, P_1 is called ***polariser*** and P_2 is called ***analyser.***

2. Identification of Given Light

For this, we pass the given light through a polaroid (called analyser), rotate the polaroid about the incident light and examine the emergent light.

(*i*) If there is no change in intensity of emergent light, incident light is *unpolarised*.

(*ii*) If there is change in intensity of emergent light (with minimum not equal to zero), the incident light is partially polarized.

(*iii*) If intensity of emergent light changes (with minimum equal to zero), the incident light is *plane polarized* or *linearly polarized*.

3. Production of Plane Polarized Light

(*i*) *Plane of vibration* : "The plane containing the direction of vibration and direction of propagation of light is called the *plane of vibration*."

(*ii*) *Plane of polarization* : "The plane containing the direction of propagation of light, but containing no vibrations, is called the plane of polarization or the plane perpendicular to the plane of vibration is called the *plane of polarization*."

4. Production and Detection of Polarized Light

The different methods commonly used for the production of plane polarized light may be classified under the following heads:

(*i*) Polarization by reflection
(*ii*) Polarization by refraction
(*iii*) Polarization by double refraction
(*iv*) Polarization by selective absorption
(*v*) Polarization by scattering

(i) Plane Polarized Light or Linearly Polarized Light: In order to produce plane polarized light, a beam of unpolarized monochromatic light is passed through a *Nicol prism*. As a beam enters the Nicol prism, it is split up into ordinary and extraordinary components. The ordinary component is totally internally reflected at the Canada - balsam layer and is absorbed; while the extraordinary component passes through the Nicol prism. The emergent light is *plane polarized* having its vibrations parallel to the shorter diagonal of end face of the Nicol.

Detection: To detect plane polarized light, it is passed through another Nicol prism which is rotated gradually about the direction of propagation of light. *If the intensity of the light*

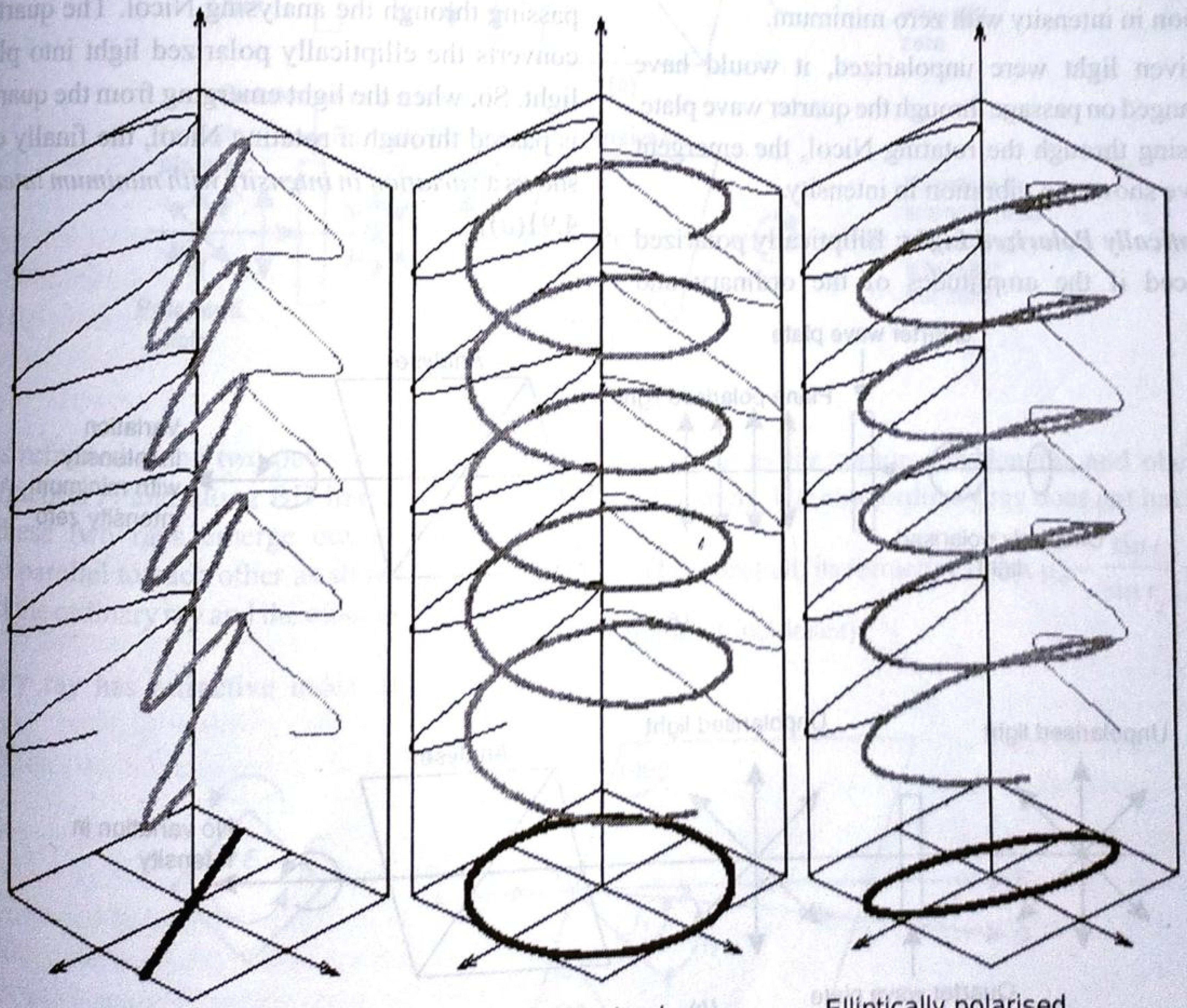


Fig. 4.90

emerging from the rotating Nicol varies with zero minimum, the light is plane polarized.

***(ii) Circularly Polarized Light*:** Circularly polarized light is produced if the amplitudes of the ordinary and extraordinary rays are equal and there is a phase difference of $\pi/2$ or a path difference of $\lambda/4$ between them.

For the purpose, the ordinary monochromatic light is passed through a Nicol prism. The light emerging from Nicol prism is plane polarized. This plane polarized light is then allowed to fall *normally* on a quarter wave plate such that the vibrations in the incident plane polarized light makes an angle of 45° with the optic axis of the plate. When this condition is satisfied, the plane polarized light on entering this quarter wave plate is split up into ordinary and extraordinary components having equal amplitude and period. The light emerging from the quarter wave plate is circularly polarized.

Detection: The circularly polarized light, when observed through a rotating Nicol, shows no variation in intensity. The same is observed when ordinary unpolarized light is viewed through a rotating Nicol.

In this respect, circularly polarized light resembles the unpolarized light. Hence, in order to detect circularly polarized light, it is first passed through a quarter wave plate which converts the circularly polarized light into plane polarized light. When this emergent light is viewed through a rotating Nicol, it shows a variation in intensity with zero minimum.

If the given light were unpolarized, it would have remained unchanged on passage through the quarter wave plate. Hence, on passing through the rotating Nicol, the emergent light would have shown no vibration in intensity.

***(iii) Elliptically Polarized Light*:** Elliptically polarized light is produced if the amplitudes of the ordinary and extraordinary rays are unequal and there is a phase difference of $\pi/2$ or a path difference of $\lambda/4$ between them.

For the purpose, the ordinary light is passed through a Nicol prism. The light emerging from the Nicol prism is plane polarized. This plane polarized light is then allowed to fall normally on a quarter wave plate such that the vibrations in the plane polarized incident light makes an angle θ ($\theta \neq 0$, 45°, 90°) with the optic axis of the plate. When this condition is satisfied, the plane polarized light on entering the quarter wave plate splits up into ordinary and extraordinary components having *unequal amplitudes* and equal periods. The light emerging from quarter wave plate is *elliptically polarized.*

***Detection*:** When elliptically polarized light is observed through a rotating Nicol, the variation in intensity is observed with minimum intensity not zero. The intensity is maximum, when the principal section of the analysing Nicol is parallel to the major axis of the elliptical vibration and minimum when parallel to the minor axis. The same is observed when partially plane polarized light is passed through a rotating Nicol.

In this respect, elliptically polarized light resembles the partially plane polarized light. In order to detect the elliptically polarized light, it is passed through a quarter wave plate before passing through the analysing Nicol. The quarter wave plate converts the elliptically polarized light into plane polarized light. So, when the light emerging from the quarter wave plate is passed through a rotating Nicol, the finally emergent light shows a *variation in intensity with minimum intensity zero* [Fig. 4.91(a)].

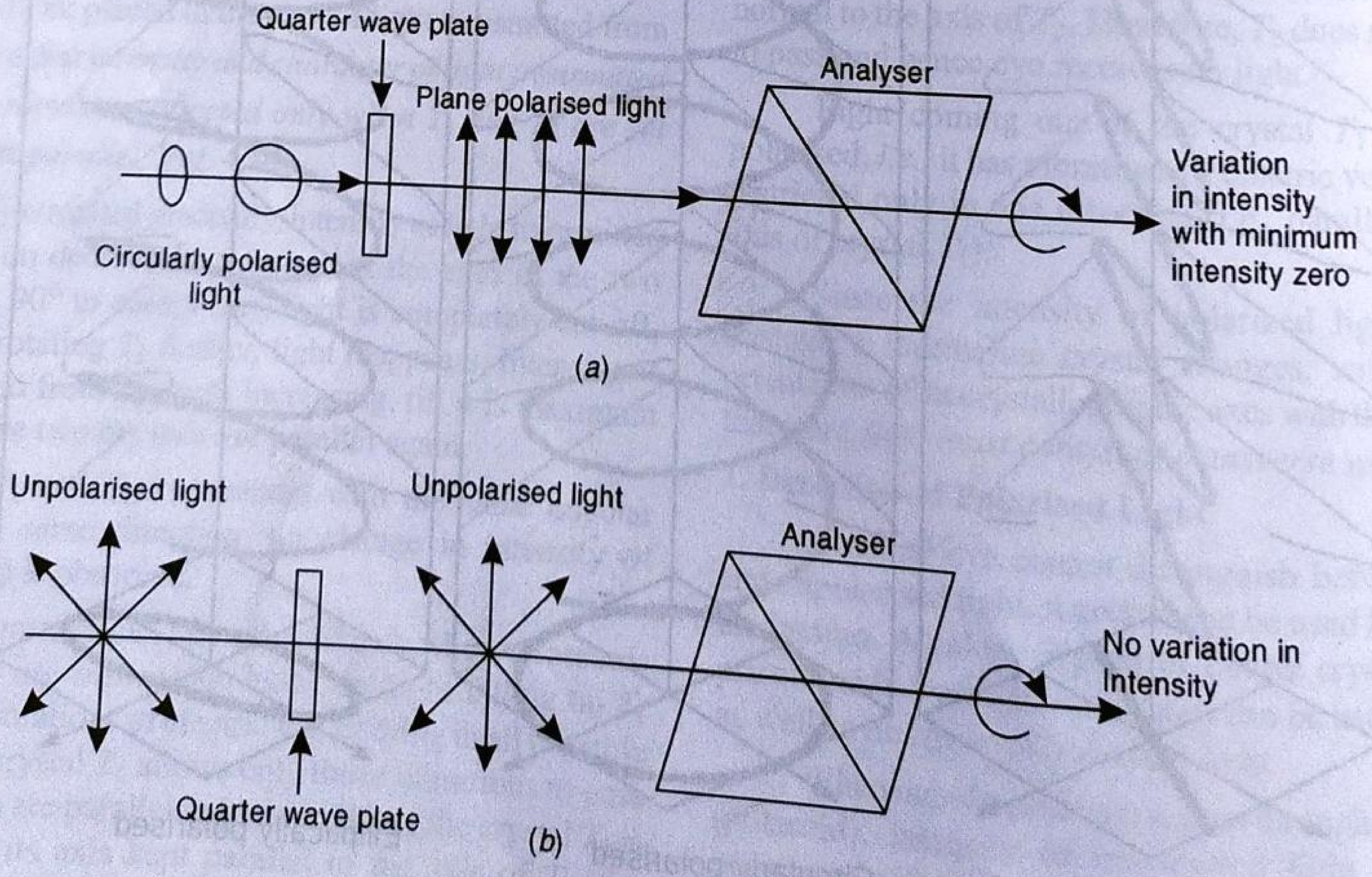


Fig. 4.91

However, if the light were partially plane polarized, it would have remained unchanged by the quarter wave plate. Hence, on passing through the rotating Nicol, the emergent light would have shown in *variation in intensity with minimum intensity (not zero)*. [Fig.4.91(*b*)].

4.17 DOUBLE–REFRACTION

When light passes through all transparent crystal except for those belonging to the cubic system, a phenomenon is observed called double refraction. **Erasmus Bartholinu**, a Danish Scientist, in 1669 discovered that when a ray of ordinary light is incident on a crystal of calcite or Iceland Spar (crystalline calcium carbonate), it splits up into two refracted rays in plane of one as in case of glass. The crystals having this property are said to be *doubly refracted crystals* and the phenomenon is called *double refraction*. This phenomenon is a direct consequence of anisotropic nature of the crystal, and is shown in addition to calcite crystal by quartz too.

Doubly refracting crystals are divided into *uniaxial* and *biaxial* ones. In uniaxial crystals, one of the refracted rays obeys the conventional law of refraction in particular, it is in the same plane as the incident ray and a normal to the refracting surface: This ray is called an ordinary ray and is designated by the symbol O. For the other ray, called an *extraordinary ray* (designated by *e*). The ratio of sines of angle of incidence and the angle of refraction does not remain constant when the angle of incidence varies. Even upon normal incidence of light on a crystal, an extraordinary ray, generally speaking, deviates from a normal uniaxial crystals have a direction along which ordinary and extraordinary rays propogate without separation and with the same velocity. This direction's called *optical axis* of the crystal.

A plane passing through an optical axis is called a *principal section* or a *principal plane* of the crystal. In some crystals one of the rays is absorbed to a greater extent than the other. This phenomenon is called **dichroism**. Let a ray of ordinary light be incident on the calcite making an angle of

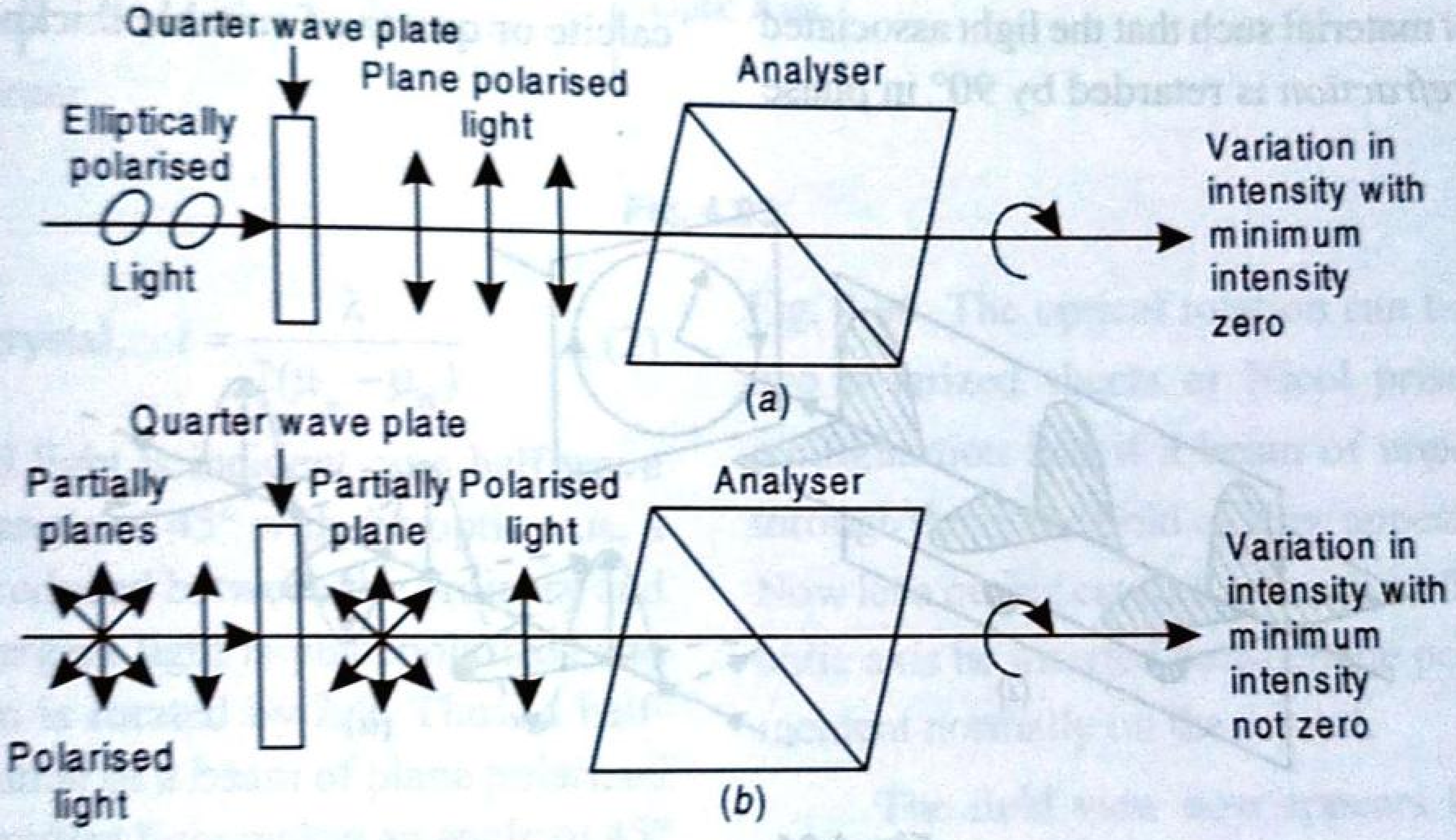


Fig. 4.92

incidence *i*, it is refracted along two paths along BC, having the angle of refraction r_2 and along BD having the angle of refraction r_1, these two rays emerge out at CE and OD respectively and parallel to each other as shown in Fig. 4.93. The one is called the ordinary ray and the other as extraordinary ray. The ordinary ray has refractive index $\mu_a = \dfrac{\sin i}{\sin r_1} = a$

(constant as for certain wavelengths and obeys the laws o[f] refraction). The extraordinary ray does not have the refractiv[e] index constant, its refractive index $\mu_e = \dfrac{\sin i}{\sin r_2}$ (varies with th[e] angle of incidence).

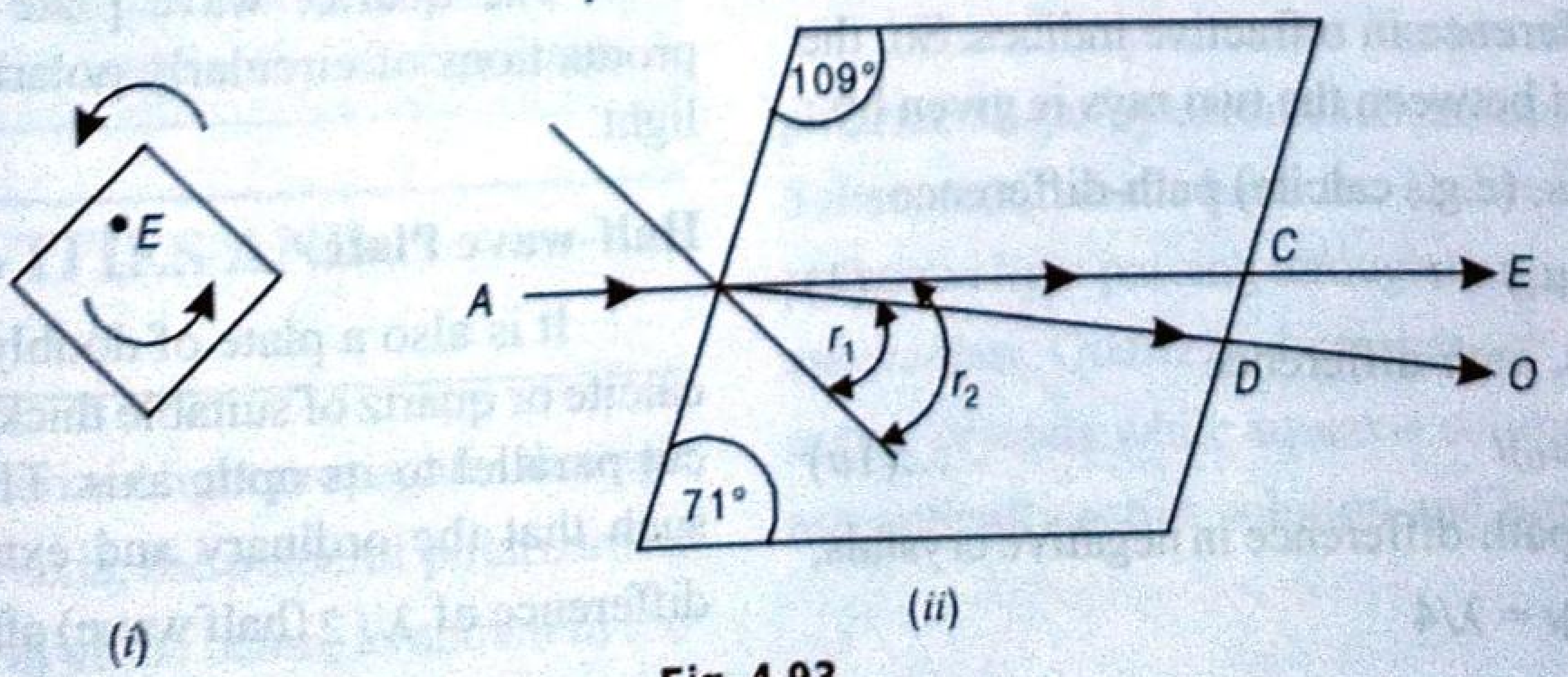


Fig. 4.93

Scanned by TapScanner

In case of calcite $\mu_0 > \mu_e$, therefore the velocity of light for the ordinary ray inside the crystal will be less as compared to the velocity of light for the extraordinary ray. As μ_e is not constant, it varies with the angle of incidence, the velocity of the extraordinary ray is different in different directions, *i.e.*, it depends on the angle of incidence.

It has been found that the rays, ordinary as well as extraordinary, are plane polarized. The vibrations of the ordinary rays are perpendicular to the principal section of crystal, while the vibrations of the extraordinary rays are in the plane of the principal section of the crystal. Thus, we can say that both rays are plane polarized, their vibrations being at right angles to each other.

4.18 QUARTER WAVE PLATES AND HALF-WAVE PLATE

Quarter Wave Plate

A quarter-wave plate consists of a carefully adjusted thickness of a *birefringent* material such that the light associated with the larger *index of refraction* is retarded by 90° in phase (a quarter wavelength) with respect to that associated with the smaller index. The material is cut so that the optic axis is parallel to the front and back plate. Any linearly polarized light which strikes the plate will be divided into two components with different indices of refraction. One of the useful applications of this device is to convert *linearly polarized* light to *circularly polarized* light and vice versa. This is done by adjusting the plane of the incident light so that it makes 45° angle with the optic axis. This gives equal amplitude o- and e-waves. When the o-wave is slower, as in calcite, the o-wave will fall behind by 90° in phase, producing circularly polarized light.

A **quarter-wave plate** creates a quarter *wavelength* phase shift and can change *linearly polarized light* to *circular* and vice versa. This is done by adjusting the plane of the incident light so that it makes 45° angle with the fast axis. This gives equal amplitude ordinary and extraordinary waves.

It is a plate of doubly refracting uniaxial crystal of calcite or quartz of suitable thickness and whose refracting

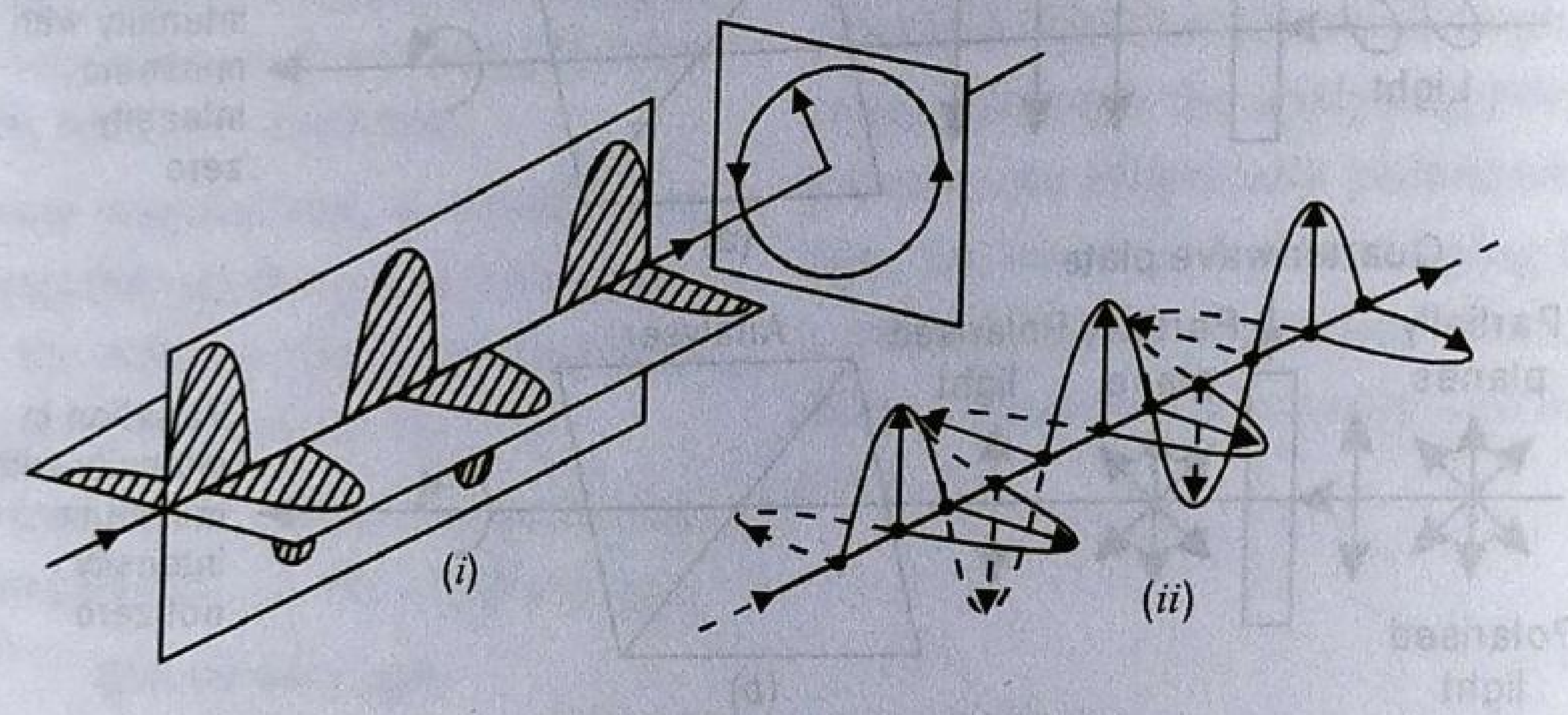


Fig. 4.94

faces are cut parallel to the direction of optic axis, so that it produces a path-difference of $\lambda/4$ between the ordinary and extraordinary rays of light. Let the thickness of the plate be 't' and the refractive indices for the ordinary and extraordinary rays are μ_0 and μ_e respectively. The plane polarized light is incident perpendicular to the surface of the plate. The ordinary and extraordinary rays travel along the same direction but with different speed due to difference in refractive indices. So, the path difference introduced between the two rays is given by :

For negative crystals, (e.g., calcite) path-difference

$$= (\mu_0 - \mu_e) \times t \qquad \ldots(1)$$

For positive crystals, path-difference

$$= (\mu_e - \mu_0)t \qquad \ldots(1a)$$

Thus, to introduce a path-difference in negative crystals,

$$(\mu_0 - \mu_e)t = \lambda/4$$

or, $$t = \frac{\lambda}{4(\mu_0 - \mu_e)} \qquad \ldots(2)$$

And to introduce a path-difference in positive crystals,

$$t = \frac{\lambda}{4(\mu_e - \mu_0)} \qquad \ldots(3)$$

The quarter wave plate finds its applications in the productions of circularly polarized and elliptically polarized light.

Half-wave Plate

It is also a plate of doubly reflecting uniaxial crystal of calcite or quartz of suitable thickness with its refracting surface cut parallel to its optic axis. The thickness 't' of the plate is such that the ordinary and extraordinary rays have a path-difference of $\lambda/2$ (half wave) after passing through the crystal.

For negative crystals, path-difference = $(\mu_0 - \mu_e)t$ and for positive crystals, path difference = $(\mu_e - \mu_0)t$.

Path-difference of half wave in a negative crystal between the two rays is,

$$(\mu_0 - \mu_e)t = \frac{\lambda}{2} \quad \text{or,} \quad t = \frac{\lambda}{2(\mu_0 - \mu_e)} \qquad \ldots (1)$$

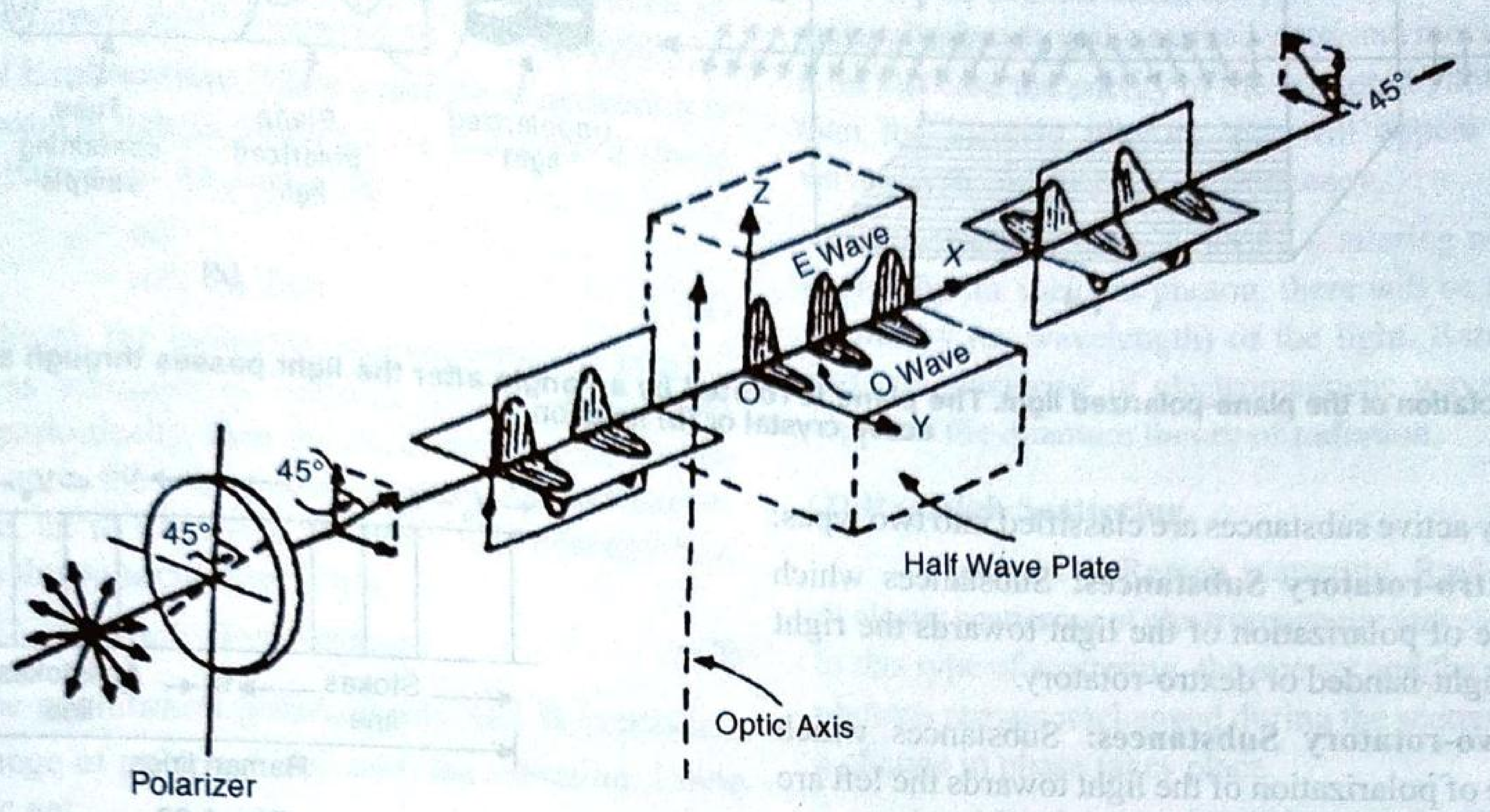


Fig. 4.95

And in a positive crystal, $t = \dfrac{\lambda}{2(\mu_e - \mu_0)}$ $\qquad \ldots (2)$

When plane polarized light is incident on a half wave plate such that it makes an angle of 45° with the optic axis, a path difference of $\lambda/2$ is introduced between the ordinary and extraordinary rays. The emergent light is planepolarized and the direction of polarization is rotated by $\lambda/2$. Thus, a half-wave plate rotates the azimuthal of a beam of plane polarized light by $\pi/2$, provided the incident light makes an angle of 45° with the axis of the half wave plate.

Polarizing Sheets
Polaroid or polarizing sheet is a trade name for a plastic dichroic absorbing material used in sheet form and the best known of which is tourmaline. Tourmaline is the original dichoric polarizer. Polaroid is the name of the plastic sheet invented by **Land** and is called *polarising sheet.* Tourmaline is a dichroic crystal which was used as a polizer or polarizing sheet. Polaroid is a doped plastic used for the same purpose.

4.19 OPTICAL ACTIVITIES AND APPLICATIONS

When a beam of plane polarized light propagates through a quartz crystal along the optic axis, the plane of polarization steadily turns about the direction of the beam, as shown in the Fig. 4.96. The optical rotation can be detected as follows. If two polarized sheets or Nicol prisms are held in crossed configuration and if a beam of unpolarized light is viewed through them, the field of view appears to be completely dark. Now let a quartz crystal, cut with its faces perpendicular to the optic axis be inserted between the polarizers such that light is incident normally on the crystal.

The field view now appears hit up indicating that the light is not cut-off by the analyser. In order to cut off the transmitted light, we find that the analyser is to be rotated through a certain angle. The experiment establishes that the plane polarized light produced by the polarizer remains plane-polarized while passing through the quartz crystal, but the plane of polarization is rotated through an angle. This angle is the angle through which the analyser is rotated in order to cut off the light totally.

The ability to rotate the plane of polarization of plane polarized light by certain substances is called *optical activity.* Substances, which have the ability to rotate the plane of the polarized light passing through them, are called *optically active substances.* Quartz and cinabar are the examples of optically active crystals while aqueous solutions of sugar, tartaric acid are optically active solution and liquid.

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