C 4T

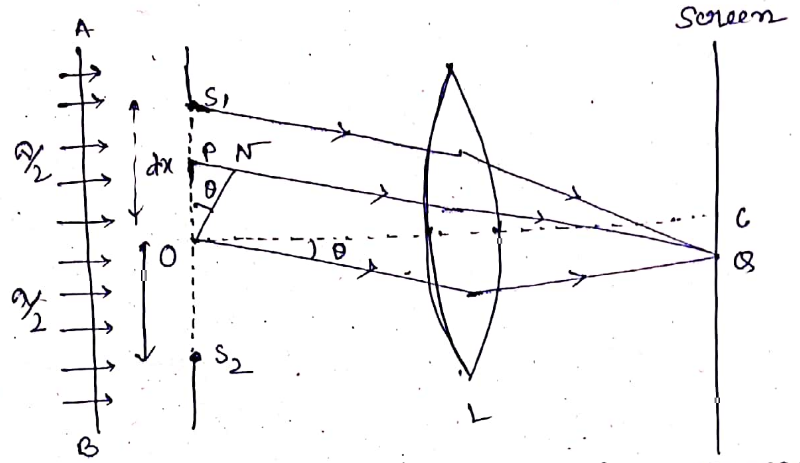
WAVES AND OPTICS

FRAUNHOFER DIFFRACTION

FOR 2ND SEM, PHYSICS (HONS.)

Fraunhoffer Diffraction :

• Single slit diffraction pattern :



Let a plane wave front AB of a monochromatic light of wavelength λ propagating normal to the slit S_1S_2. According to Hygens- Fresnel's principle each point on the wave front regarded as the source of secondary spherical wavelets.

The wavelets travelling normal to the slit are brought to focus by convex lens L, on the screen C. the wave travels at 'θ' angle with normal and brought into focus on 'Q'. So, intensity will find at 'Q'

Let the complex light disturbence at any instant $= Ae^{i\omega t}$, A = Amplitude, ω = circular frequency of the wave.

So, the phase diff. produced between the waves at Q from O and P at a distance x from O is given by,

$$\frac{2\pi}{\lambda} \times PN = \frac{2\pi}{\lambda} \cdot x \sin\theta = lx$$

$l = \frac{2\pi}{\lambda} \sin\theta$

Now. disturbance at Q due to secondary waves from P will be proportional to $e^{i(\omega t - lx)}$, the disturbence at Q due to diffraction element dx

$$dy = CA\,dx\, e^{i(\omega t - lx)}$$

$A \propto dx$
C = proportion Const.

if a is the width of the slit then the resultant complex disturbance at Q, due to the waves coming from all the diffracting elements will be ——

$$y = \int_{-a/2}^{+a/2} CA\, e^{i(\omega t - lx)}\, dx$$

$$= CA\, e^{i\omega t}\left[\frac{e^{-ilx}}{-il}\right]_{-a/2}^{+a/2}$$

$$= CAa\, \frac{\sin\frac{la}{2}}{\frac{la}{2}}\, e^{i\omega t} \qquad \left[\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2}\right]$$

Now, intensity $I = yy^* = I_0 \dfrac{\sin^2\alpha}{\alpha^2}$ | $I_0 = (CAa)^2$, $\alpha = \dfrac{la}{2} = \dfrac{\pi}{\lambda} a\sin\theta$

∴ <u>To find Maxima and Minima</u>:

Let, $\dfrac{dI}{d\alpha} = 0$

$$\frac{2\sin\alpha\cos\alpha}{\alpha^2} - \frac{2\sin^2\alpha}{\alpha^3} = 0$$

$$\sin\alpha \cdot (\alpha\cos\alpha - \sin\alpha) = 0$$

So, $\sin\alpha = 0$, $\dfrac{d^2I}{d\alpha^2} = +ve$ for | $\alpha = \tan\alpha$, $\dfrac{d^2I}{d\alpha^2} = -ve$

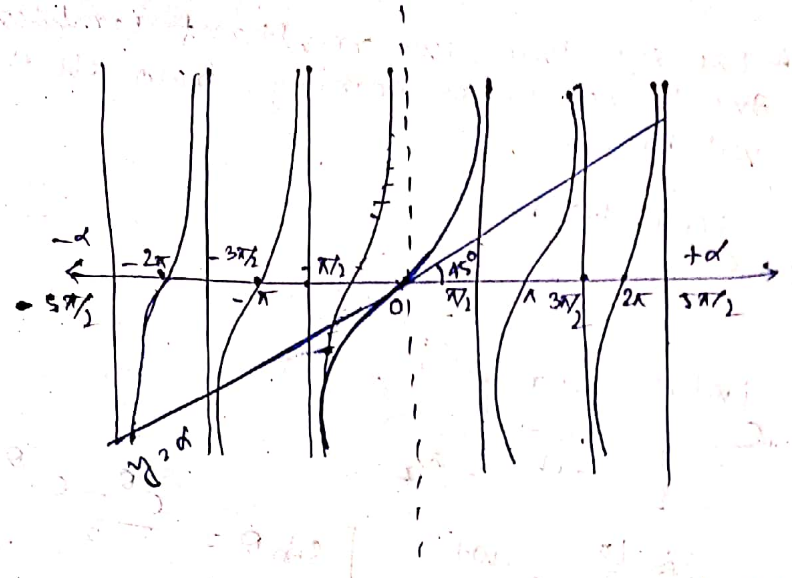
So, minima, $\sin\alpha = 0$

$$\alpha = m\pi, \quad m = \pm 1, \pm 2 \ldots$$

$$\boxed{a\sin\theta = m\lambda}$$

So, $m = 0$ excluded, because for this $\alpha = 0$, and since

$\lim_{\alpha \to 0} \dfrac{\sin\alpha}{\alpha} = 1$, its actually gives a maximum.



So, $\alpha = 0$ given principal maxima, others gives secondary maxima.

the intensity of principal maxima is

- $I = \lim_{\alpha \to 0} I_0 \cdot \frac{\sin^2\alpha}{\alpha^2} = I_0$

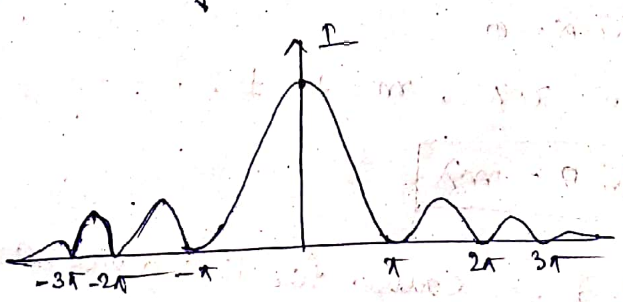
- The intensity of first secondary maximum is

$$I_1 \approx I_0 \cdot \frac{\sin^2 3\pi/2}{(3\pi/2)^2} = \frac{4}{9\pi^2} \cdot I_0$$

- intensity of second secondary maxima

$$I_2 \approx I_0 \cdot \frac{\sin^2 5\pi/2}{(5\pi/2)^2} = \frac{4}{25\pi^2} \cdot I_0$$

* intensity distribution in the diffraction pattern due to a single slit —



If the angle of diffraction θ_1 for the first minima on either side of central maxima,

$$a \sin\theta_1 = \lambda$$

$$\theta_1 \approx \sin\theta_1 \approx \lambda/a$$

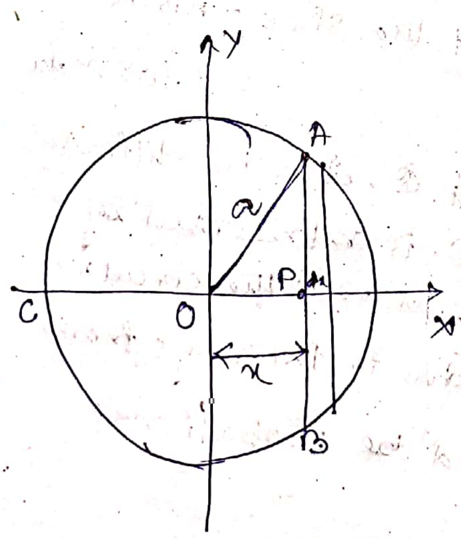
So, angular width of the principal maxima $= 2\theta_1$ is inversely proportional to width (a) of the slit.

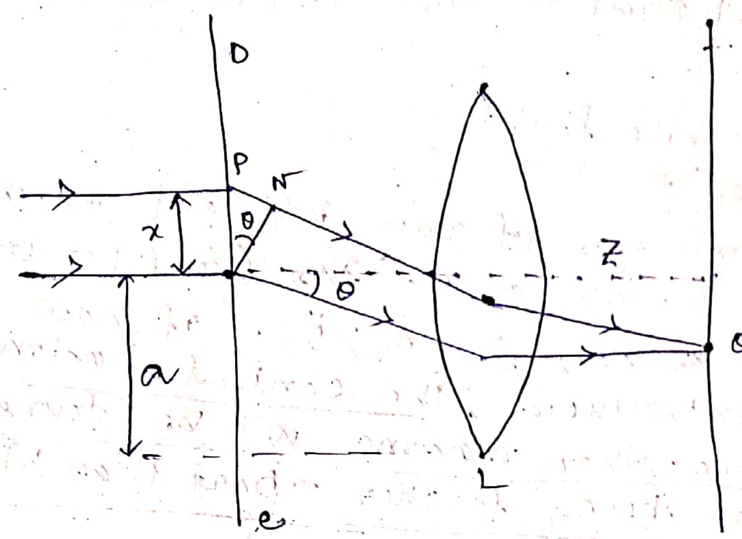
Effect due to white light:

The condition for minima of rays incident normally on the slit is: $a\sin\theta = m\lambda$ or $\theta = \lambda/a$ (for 1st order dark band). As $\lambda_r > \lambda_v$ So, $\theta_r > \theta_v$. If now white light is introduced, the central maxima becomes white, while others maxima will be coloured, and red maxima being farther apart than blue

Fraunhoffer diffraction in a circular Aperture:

Suppose a parallel beam of light of wavelength λ is incident normally on a circular aperture of radius a. Here also Hygens - Fresnel's theory applied.



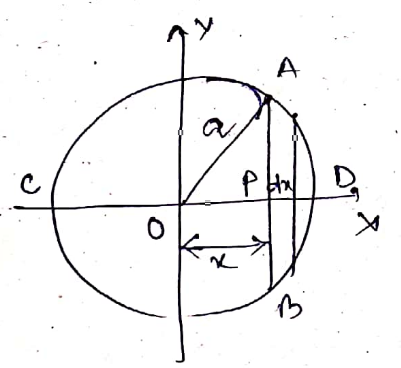


Let us consider rays diffracted at angle θ with normal OZ to the aperture.

∴ Resultant intensity will be calculated at the point Q.

. . . ~~In the focal plane~~ .

• The phase diff between the rays which are diffracted at an angle θ from the origin O and form a point P on the horizontal diameter (CD) at a distant x from the origin is. $\delta = \frac{2\pi}{\lambda}(OM) = \frac{2\pi}{\lambda} . x \sin\theta$



Let us consider a strip AB of the circular aperture at a distance x from the origin 'O' having its width dx, the area of the strip = AB dx

$= 2\sqrt{a^2-x^2}\,dx$

Now, complex disturbance at Q, due to diffracted rays at an angle 'θ' from O. is represented by $Ae^{i\omega t}$, where A is the Amplitude and ω is the circular frequency. The distrubence at Q due to the rays from P diffracted at an angle θ would be proportional to $e^{i(\omega t-\delta)}$

So, complex disturbence due to the rays proceeding from strip AB and diffracted at θ angle

$$dy = C.A.e^{i(\omega t-\delta)} . 2\sqrt{a^2-x^2}\,dx$$

So, total disturbance due to entire aperture and diffracted at an angle $= \theta$,

$$y = 2CA e^{i\omega t} \int_{-a}^{+a} e^{-i \cdot \frac{2\pi}{\lambda} \cdot x \sin\theta} \cdot \sqrt{a^2 - x^2}\, dx$$

the real amplitude of the disturbance is

$$R = 2CA \int_{-a}^{+a} \sqrt{a^2 - x^2} \cdot \cos\left(\frac{2\pi}{\lambda} x \sin\theta\right) dx$$

putting, $x = au$; $\frac{2\pi}{\lambda}(a\sin\theta) = m$ we get,

$$\boxed{R = 2CAa^2 \int_{-1}^{+1} \sqrt{1-u^2} \cos(mu)\, du}$$

Now, $\int_{-1}^{+1} \sqrt{1-u^2} \cdot \cos(mu)\, du$ is a Bessel function of first order whose value can be written as,

$$\int_{-1}^{+1} \sqrt{1-u^2} \cdot \cos mu \cdot du = \pi \frac{J_1(m)}{m}$$

$J_1(m) =$ Bessel function of 1st order

So, $R = 2CAa^2\pi \frac{J_1(m)}{m} = CA(\pi a^2)\left[\frac{2J_1(m)}{m}\right]$

So, the intensity at the given point is

$$\boxed{I = R^2 = I_0 \left[\frac{2J_1(m)}{m}\right]^2}$$

$I_0 = (CA\pi a^2)^2$

Now as θ increases, m also increases, but $\frac{J_1(m)}{m}$ alternately increases and decreases.

Scanned with CamScanner

Let, $I = I_0 \left[\frac{2J_1(m)}{m}\right]^2$ $I_0 = [CA\pi a^2]^2$

- **principal Maxima :**

when, $m=0$, $\frac{J_1(m)}{m}$ becomes indeterminate.

but $\lim_{m\to 0} \frac{J_1(m)}{m} = \frac{1}{2}$, hence intensity, $I = I_0$

Now, $m \to 0$, $\theta \to 0$ so the maximum intensity occours at the geometrical image of the source.

→ The principal maximum is a central bright disc called Airy's disc, whose intensity $\propto$ Area of the aperture (πa^2)

- **Seconday minima :**

The intensity would be zero at $J_1(m) = 0$ except at the point $m=0$, which will give the central maxima.

The first minimum which surrounds the central maximum, will corresponds to the first zero of the function $J_1(m)$ thus.

$m = 1.22\pi$

$2a \sin\theta = 1.22\lambda$

$\sin\theta = \frac{1.22\lambda}{D}$ $| D = 2a$

Now, $\sin\theta' = \frac{2.233\lambda}{D}$, $\sin\theta'' = \frac{3.238\lambda}{D}$

Now, angular radius of the 1st dark ring $= \theta = \frac{1.22\lambda}{D}$

Linear " " " $\rho = \frac{1.22\lambda . f}{D}$

[f = focal length of the lens]

Secondary Maxima :

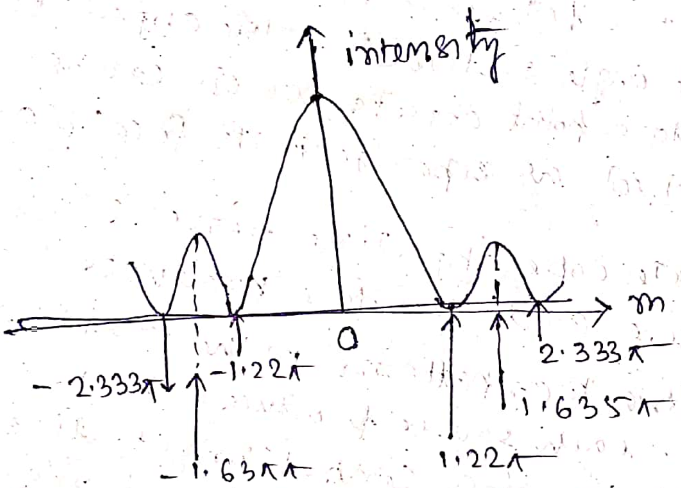
Secondary maxima would be obtaine when $\frac{d}{dm}\left(\frac{J_1(m)}{m}\right) = 0$

2nd maxima , $m = 1.635\pi$

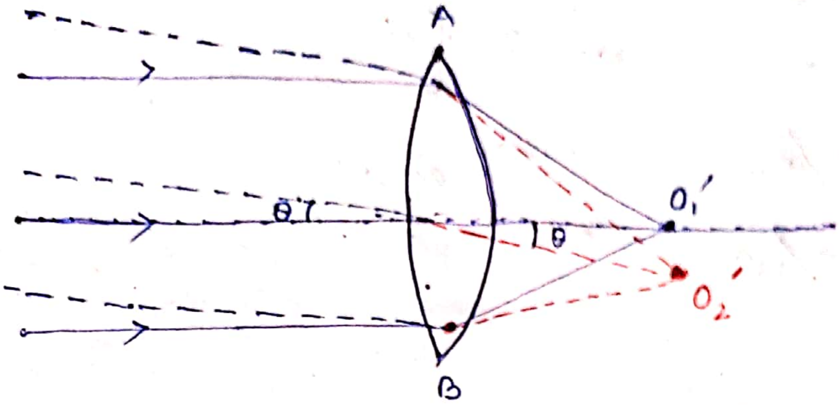
3rd " $m = 2.679\pi$

$I_2 = I_0/56$, 2nd Maxima

$I_3 = I_0/119$, 3rd



• Resolving Power of a Telescope



resolving power of a telescope is defined as the inverse of the least angle subtended at the objective by two distant close point objects which can be just distinguished as separate in its focal plane.

Let AB = D, the telescope objective serves as a circular aperture and therefore produces fraunhoffer diffraction pattern in the focal plane of the objective. each source produce a own diffraction pattern of a central bright disc surrounded by the concentric dark and bright ~~discs~~ rings.

• Let O_1' and O_2' be the common centres of the diffraction patterns due to the two sources O_1 and O_2 respectively. According to Rayleigh's criterion for just resolution the centre O_2' of the central bright disc of the source O_2 must coincide with the first dark ring in the diffraction pattern of O_1. Angular separation θ, in between O_1' and O_2' will be equal to the angular separation of the 1st minimum from the centre O_1' of the diffraction pattern of O_1. Now from theory of fraunhoffer diffraction at circular aperture of diameter D, the angular separation θ of 1st minimum from the centre of the diffraction pattern is given by .

$$\sin\theta = \frac{1.22\lambda}{D}$$

$$\boxed{\sin\theta \approx \theta \approx \frac{1.22\lambda}{D}}$$ ⟶ This is the min. angular separation between two point objects that can be just resolved by the telescope.

So, $$\boxed{R.P = \frac{1}{\theta} = \frac{D}{1.22\lambda}}$$

• <u>Relationship : R.P. to Magnifying Power of a telescope</u> :⇒

Magnifying Power $M.P = D/d$

$d = d_E$ = diameter of the pupil eye

D = Diameter of the objective . i.e. an entrance pupil

d = Diameter of exit pupil

So, $M = \frac{D}{d_E}$

• So, Limit of Resolution of Telescope ⇒ $\theta = \frac{1.22\lambda}{D}$

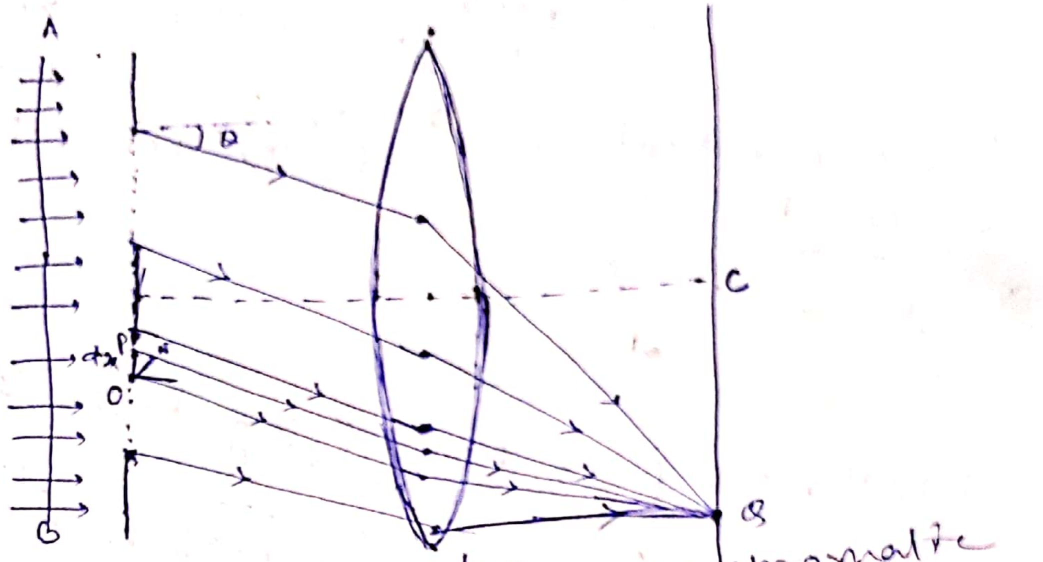
• • " " " eye ⇒ $\theta' = \frac{1.22\lambda}{d_E}$

$$\frac{\theta'}{\theta} = \frac{D}{d_E} = \text{Normal M.P. of a telescope.}$$

So, Normal Magnifying Power of Telescope • Its limit of Resolution (Telescope's Resolution) = Limit of Resolution of the eye

H.T ⟶ Similary do for M-scope .

Fraunhoffer Diffraction in a double slit :



Let, a parallel beam of mono chromatic light of wave length λ is being incident. width of both open/clear space $= a$

width of opaque space $= b$

So, the distance between any pair of corres- ponding points of the two slits is $d = a+b$.

According to Hygen's - Fresnel's principle — thus each points of the primary wave lets is the source of the secondary wave.

– The phase diff between the waves at Q comming from O and from P which is at a distance x from O is given by,

$$\frac{2\pi}{\lambda}\times PN = \frac{2\pi}{\lambda}\times x\sin\theta \quad . \quad l = \frac{2\pi\sin\theta}{\lambda}$$

Similar like Single slit

$$dy = CA\,dx\,e^{i(\omega t - ka)}$$

$C =$ Const

$A =$ Amplitude.

So the resultant complex disturbance at Q due to the both slits,

$$y = \int_{-a/2}^{+a/2} CA\, e^{i(\omega t - kx)}\, dx + \int_{d-a/2}^{d+a/2} CA\, e^{i(\omega t - kx)}\, dx$$

$$= CA \cdot \left[\frac{e^{-ilx}}{-il}\right]_{-a/2}^{+a/2} + CA e^{i\omega t}\left[\frac{e^{-ilx}}{-il}\right]_{d-a/2}^{d+a/2}$$

$$= CAa \cdot \frac{\sin \frac{al}{2}}{\frac{al}{2}} \left[1 + e^{-ild}\right] \cdot e^{i\omega t}$$

So the intensity becomes

$$I = yy^* = (CAa)^2 \cdot \frac{\sin^2 \frac{al}{2}}{\left(\frac{al}{2}\right)^2} \cdot \left(1 + e^{-ild}\right)\left(1 + e^{+ild}\right)$$

$$= (CAa)^2 \cdot \frac{\sin^2 \frac{al}{2}}{\left(\frac{al}{2}\right)^2} \left[2 + e^{ild} + e^{-ild}\right]$$

$$= (CAa)^2 \cdot \frac{\sin^2 \frac{al}{2}}{\left(\frac{al}{2}\right)^2} \left(2 + 2\cos ld\right)$$

$e^{i\theta} = \cos\theta + i\sin\theta$

$e^{-i\theta} = \cos\theta - i\sin\theta$

$$\boxed{I = 4I_0 \cdot \frac{\sin^2\alpha}{\alpha^2} \cdot \cos^2\beta}$$

where $I_0 = (CAa)^2$

$$\alpha = \frac{al}{2} = \frac{\pi}{\lambda} a \sin\theta$$

$$\beta = \frac{ld}{2} = \frac{\pi}{\lambda}(a+b)\sin\theta$$

$$I = \underbrace{4I_0 \frac{\sin^2\alpha}{\alpha^2}}_{I_1} \underbrace{\cos^2\beta}_{I_2}$$

$I_1 = 4I_0 . \frac{\sin^2\alpha}{\alpha^2}$ which gives diffraction pattern of a single slit.

$I_2 = \cos^2\beta$ which gives interference pattern of both the slits.

- For minima :

At $\frac{\sin^2\alpha}{\alpha^2} = 0$

$\sin\alpha = 0$, but $\alpha \neq 0$

So, $\alpha = m\pi$, $m = \pm1, \pm2 \dots$

So, $\boxed{a \sin\theta = m\pi}$ condition for Diffraction minima.

now, if $\cos^2\beta = 0$ $\Rightarrow \cos\beta = 0$

$\beta = \pm (2s+1)\pi/2$, $s = 0, 1, 2 \dots$

So, $\boxed{d \sin\theta = (a+b)\sin\theta = \pm (2s+1)\lambda/2}$

Condition for interference minima.

- For maxima :

D [Position of maxima due to the factor $\frac{\sin^2\alpha}{\alpha^2}$ areat $\alpha = 0$ and at values approaching $\pm 3\pi/2$, $\pm 5\pi/2$ etc.

I [for interference the maxima due to the factor $\cos^2\beta$ are given by $\beta = p\pi$

$p = 0, \pm1, \pm2, \dots$

Missing order :

If the condition for maxima of interference pattern and minima of diffraction pattern are simultaneously satisfied for a given value of θ, then interference maxima will be missing.

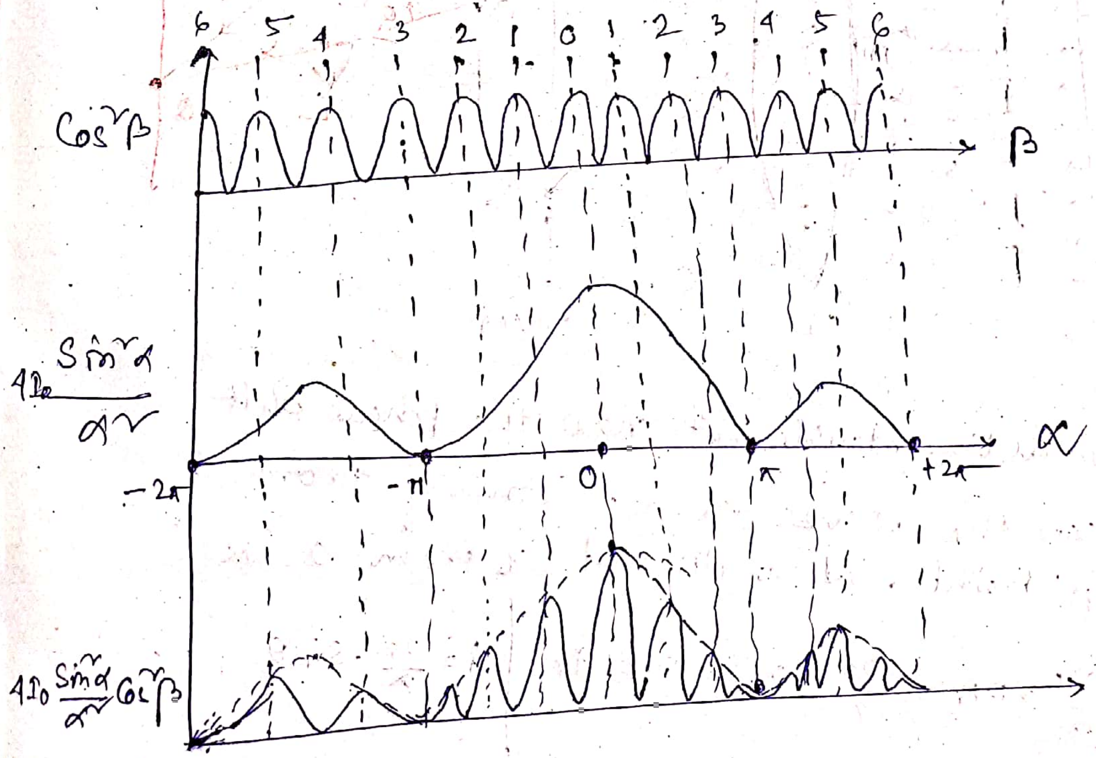
If the slit width = a is constant and separation b is varied then the distance between consecutive interference maxima changes but the diffraction pattern due to single slit remains constant.

for a given θ, $\frac{a+b}{a} = \frac{p}{m}$

So, the missing order ratio $\frac{a+b}{a}$ should be the ratio of two integers. if $\frac{a+b}{a} = \frac{2}{1}$, $a = b$, $p = 2m$, then 2, 4, 6 e.t.c. order interference maxima will absent.

Now, if $\frac{a+b}{a} = \frac{3}{1} \Rightarrow b = 2a$, $p = 3m$ then 3, 6, 9 ... order will be absent.

Complete double slit pattern :



Here $\frac{a+b}{a} = 3$

• **Diffraction Gratting**: plane diffraction gratting consists a no. of parallel and equidistant lines rulled on an optically plane and parallel glass plate by a fine dimond point. The no. of such ruled lines per mm is of the order of 100.

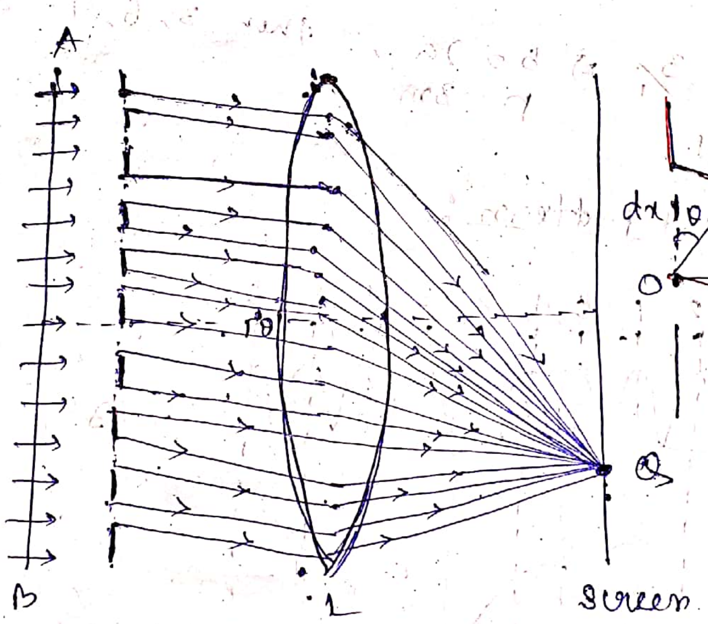
a = width of the clear space

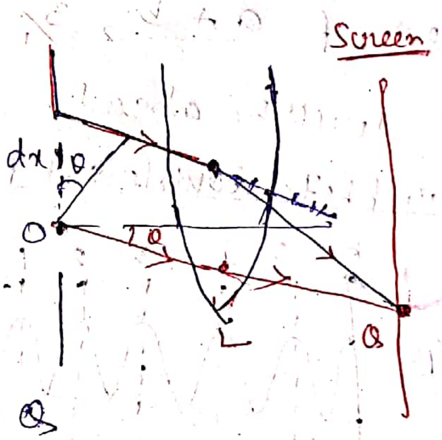
b = width of the opaque space

d = a+b = Grating Element.

→ • Two points in the consecutive clear space whose distance of separation is (a+b), are called corresponding points.

As like single slit double slit similarly;





Similar as like double slit, now the phase diff between the waves at Q. Coming from O and P' point, at a distant x from O' is

$$\frac{2\pi}{\lambda} \times PN = \frac{2\pi}{\lambda} \times x\sin\theta = \delta x \quad \Big| \quad \delta = \frac{2\pi}{\lambda} \cdot \sin\theta$$

So, the disturbence at Q due to Secondary waves from P will be proportional to $e^{i(\omega t - \delta x)}$.

∴ So, the disturbance at Q due to all (grating element

$dx \longrightarrow$ ~~2nd~~ $dy = CA\,dx\,e^{i(\omega t - \lambda x)}$

$A \to$ Amplitude
$C \to$ Const.
$A \propto dx$.

Now, the resultant complex disturbance at Q due to all the slits will be

$$y = \int_{-a/2}^{+a/2} CA\,e^{i(\omega t - \lambda x)}\,dx + \int_{d-a/2}^{d+a/2} CA\,e^{i(\omega t - \lambda x)}\,dx + \dots + \int_{(N-1)d-a/2}^{(N-1)d+a/2} CA\,e^{i(\omega t - \lambda x)}\,dx$$

where $a+b = d$ is the grating element.

$$y = CA\,e^{i\omega t}\left[\frac{e^{-i\lambda x}}{-i\lambda}\right]_{-a/2}^{+a/2} + CA\,e^{i\omega t}\left[\frac{e^{-i\lambda x}}{-i\lambda}\right]_{d-a/2}^{d+a/2} + \dots + CA\,e^{i\omega t}\left[\frac{e^{-i\lambda x}}{-i\lambda}\right]_{(N-1)d-a/2}^{(N-1)d+a/2}$$

$$= CAa\,\frac{\sin\frac{a\lambda}{2}}{\frac{a\lambda}{2}}\left[1 + e^{-i\lambda d} + \dots + e^{-i(N-1)\lambda d}\right]e^{i\omega t}$$

$$y = CAa\cdot\frac{\sin\frac{a\lambda}{2}}{\frac{a\lambda}{2}}\,\frac{\left[e^{-iN\lambda d} - 1\right]}{\left[e^{-i\lambda d} - 1\right]}\,e^{i\omega t}$$

So, the resultant intensity I at Q

$$I = yy^* = (CAa)^2\cdot\frac{\sin^2\frac{a\lambda}{2}}{\left(\frac{a\lambda}{2}\right)^2}\,\frac{\left[e^{-iN\lambda d} - 1\right]\left[e^{+iN\lambda d} - 1\right]}{\left[e^{-i\lambda d} - 1\right]\left[e^{+i\lambda d} - 1\right]}$$

$$= (CAa)^2\,\frac{\sin^2\frac{a\lambda}{2}}{\left(\frac{a\lambda}{2}\right)^2}\,\frac{\left(2 - e^{iN\lambda d} - e^{-iN\lambda d}\right)}{\left(2 - e^{i\lambda d} - e^{-i\lambda d}\right)}$$

$$= (CAa)^2\cdot\frac{\sin^2\frac{a\lambda}{2}}{\left(\frac{a\lambda}{2}\right)^2}\cdot\frac{\left[2 - 2\cos N\lambda d\right]}{\left[2 - 2\cos\lambda d\right]}$$

$$\therefore \boxed{I = I_0 \cdot \frac{\sin^2\alpha}{\alpha^2} \cdot \frac{\sin^2 N\beta}{\sin^2\beta}} \qquad \left[e^{\pm i\theta} = \cos\theta \pm i\sin\theta \right]$$

Where,

$$I_0 = (CA\,a)^2$$

$$\alpha = \frac{\alpha l}{2} = \frac{\pi}{\lambda} \cdot a \sin\theta$$

$$\beta = \frac{\pi}{\lambda} d \sin\theta = \frac{\pi}{\lambda}(a+b)\sin\theta$$

$$I = \underbrace{I_0 \cdot \frac{\sin^2\alpha}{\alpha^2}}_{I_1} \cdot \underbrace{\frac{\sin^2 N\beta}{\sin^2\beta}}_{I_2}$$

$I_1 = I_0 \cdot \frac{\sin^2\alpha}{\alpha^2}$ = Gives diffraction pattern of a single slit.

$I_2 = \frac{\sin^2 N\beta}{\sin^2\beta}$ = Gives interference pattern of the diffracted light beams from N slits.

- Now putting $N = 2$, we get double slit intensity.

- **principal Maxima :**

if the slit width is very small and observation is confined to the neighbourhood due to the factor $\frac{\sin^2\alpha}{\alpha^2}$ is small & under the condition the maxima will be slowly controlled by the factor

$I_2 = \frac{\sin^2 N\beta}{\sin^2\beta}$ So, $\beta = m\pi$, $\beta = 0, \pm 1, \pm 2$.

$\boxed{(a+b)\sin\theta = m\lambda}$ principle maxima Diffraction.

now $\beta = m\pi$, $I_2 = \frac{0}{0}$, so interminate, in the limit $\beta \to m\pi$ we get maximum value for I_2

$$\lim_{\beta \to m\pi} \frac{\sin N\beta}{\sin\beta} = \lim_{\beta \to m\pi} \frac{N\cos N\beta}{\cos\beta} = N \quad \text{[L Hospital rule]}$$

So, $I_2 = N^2$ and $I = I_{pm} = I_0 \frac{\sin^2\alpha}{\alpha^2} \times N^2 = I_1 \times N^2$

- Thus the intensity of principal maxima increases as the no. of slit's (N) increases. but due to the presence of the factor $\frac{\sin^2\alpha}{\alpha^2}$, whose value decreases with increase the angle of diffraction (θ). the intensity of principal maxima decreases with the increase in the order no. of bands.

- Condition for secondary minima and maxima...

$I_2 = \frac{\sin^2 N\beta}{\sin^2\beta}$ depends on β for maxima or minima

$$\frac{dI_2}{d\beta} = 0$$

$$\Rightarrow \frac{dI_2}{d\beta} = \frac{2N\sin N\beta\cos N\beta}{\sin^2\beta} - \frac{2\sin^2 N\beta\cos\beta}{\sin^3\beta}$$

$$= 2 \cdot \frac{\sin^2 N\beta}{\sin^2\beta}\left[N\cot N\beta - \cot\beta\right]$$

Hence, for maxima or minima, either $\frac{\sin N\beta}{\sin\beta} = 0$
$\Rightarrow N\cot N\beta = \cot\beta$

•• Secondary maxima:

$N\cot N\beta = \cot\beta$ makes $\frac{dI_2}{d\beta} = 0$

and, for maxima, $\frac{dI_2}{d\beta} = -ve$

So, for intensity of the ~~2ndd~~ secondary maxima

$$N\cot N\beta = \cot\beta$$

$$N^2 \cdot \frac{\cos^2 N\beta}{\cos^2\beta} = \frac{\sin^2 N\beta}{\sin^2\beta}$$

or, $$\frac{N^2(1-\sin^2 N\beta)}{1-\sin^2\beta} = \frac{N^2\sin^2 N\beta}{N^2\sin^2\beta} = \frac{N^2}{1+(N^2-1)\sin^2\beta} \quad \left[\because \frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}\right]$$

So, the intensity of secondary maxima is given by,

$$I_{sm} = I_1 \times \frac{N^2}{1+(N^2-1)\sin^2\beta} = \frac{I_{pm}}{1+(N^2-1)\sin^2\beta}$$

18

or, $\dfrac{I_{sm}}{I_{pm}} = \dfrac{1}{1+(N^2-1)\sin^2\beta}$

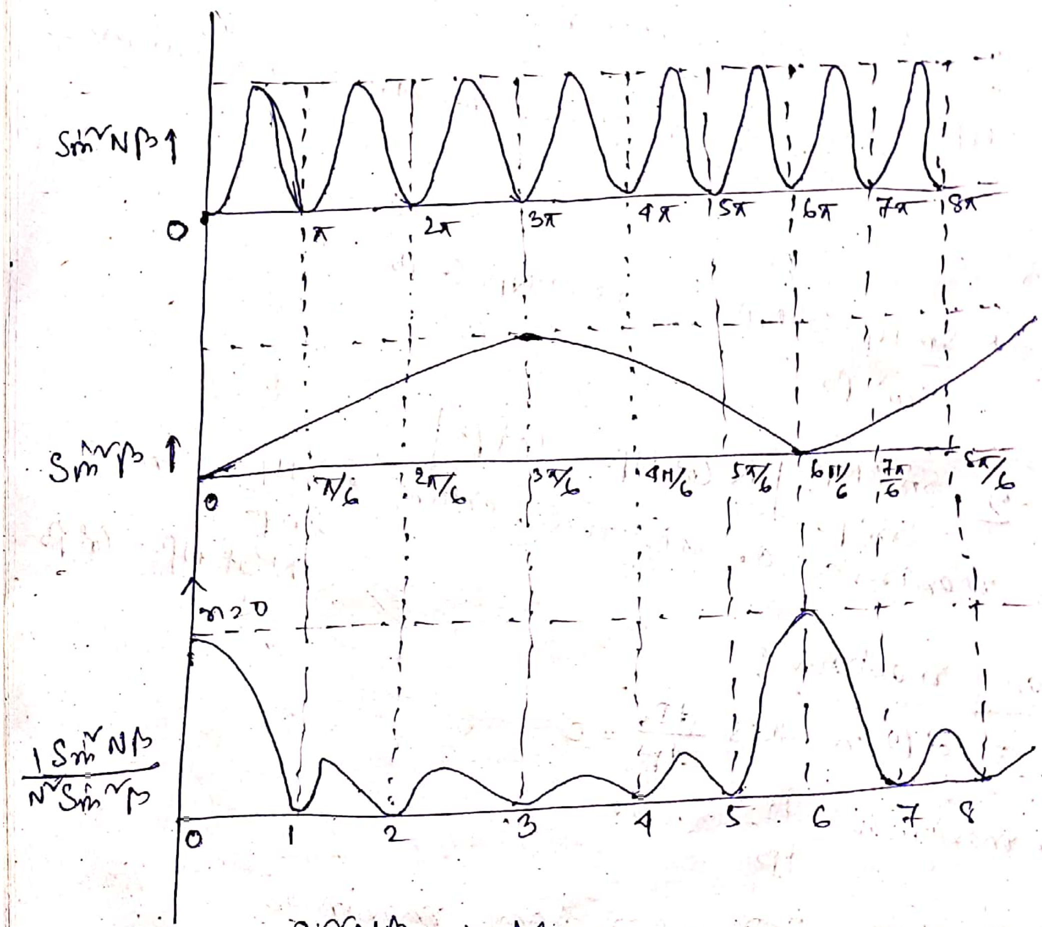
- Secondary minima :

Let, $\sin N\beta = 0$ but $\sin\beta \neq 0$, then $\dfrac{\sin^2 N\beta}{\sin^2\beta} = 0$

hence intensity becomes 0. So,

$$N\beta = \pm s\pi$$

$$(a+b)\sin\theta = \pm \frac{s}{N}\lambda$$



$N=6$, $\dfrac{\sin^2 N\beta}{\sin^2\beta}$ is Max.

Absent spectra and Ghosts in Diffraction Gratings:

for m th order principal maxima in the direction θ we have,

$$(a+b)\sin\theta = m\lambda$$

suppose the value of a is such that sth order diffraction minimum occours in the same direction θ, then

$$a\sin\theta = s\lambda$$

if the two condition satisfy simultaneously then 'm' th order principle maxima will be absent from the spectra.

$$\frac{a+b}{a} = \frac{m}{s}$$

$a = b \Rightarrow m = 2s$, 2, 4, 6 order will be absent.

Ghosts: In an ideal grating rullings should be equally spaced. But in practically there are remains some errors in the rullings. If the error is random the grating gives a continuous background illumination. if the error is progressive in nature, the spectral lines becomes sharper in planes, which are different from the focal plane of the optical system.

This most common error is periodic, so defects in the driving mechanism of the rulling machine. It gies rise to false lines accompanying the principle maxima of ideal grating. The additional false lines are called Ghosts.

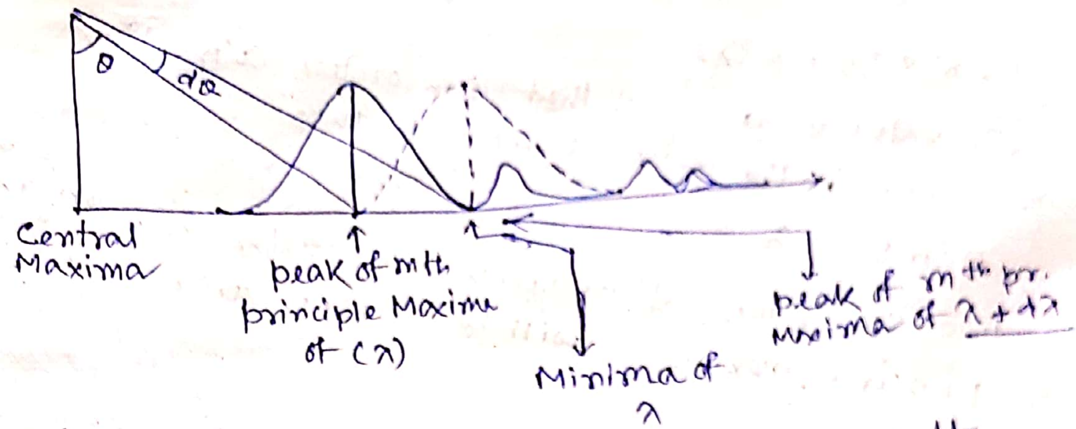
Angular dispersive power of a Grating:

$$\frac{d\theta}{d\lambda} = \frac{m}{(a+b)\cos\theta}$$

m = order no.

a+b = Grating element.

Resolving Power of a Grating :



Resolving power of a grating measures its ability to distinguish two close spectral lines and is defined by $\lambda/d\lambda$ is the smallest wave length difference for which spectral lines can be just resolved at wavelength λ. According to Rayleigh two spectral lines of wavelength λ and $\lambda+d\lambda$ are said to just resolved if the pr. maxima corresponds to the wavelength $\lambda+d\lambda$ falls on the first minimum of wavelength λ and vice-versa. Again, the angular separation (dθ) between the two pr. Max. of two spectral lines in a given order 'm' must be equal to half the angular width of the principal maxi. of λ.

Let the parallel rays consist wave length λ and $\lambda+d\lambda$ incident on the grating of grating element $a+b$ at angle θ and $\theta+d\theta$.

$$\Rightarrow \quad (a+b)\sin\theta = m\lambda$$

$$d\theta = \frac{m\,d\lambda}{(a+b)\cos\theta}$$

which gives angular separation ($d\theta$) between the two m th principle maxima corresponding to λ and $\lambda + d\lambda$, So,

$$N(a+b)\sin\theta = Nm\lambda$$

→ N = total no. of slits in the grating.

$$N(a+b)\sin\theta = 0, N\lambda, 2N\lambda \ldots$$ which gives direction of 0, 1st, 2nd. order of principal maxima for the wavelength λ

So, the position of the minimum intensity

$$N(a+b)\sin\theta = \lambda, 2\lambda \ldots (N-1)\lambda, (N+1)\lambda, (N+2)\lambda \ldots$$ mth principal maxim

So, the 1st minimum after the mth principal maxima

$$N(a+b)\sin(\theta + d\theta) = Nm\lambda + \lambda$$

$$\Rightarrow \frac{\sin(\theta + d\theta)}{\sin\theta} = \frac{Nm\lambda + \lambda}{Nm\lambda}$$

$$\Rightarrow \frac{\sin\theta + d\theta \cdot \cos\theta}{\sin\theta} = 1 + \frac{1}{Nm}$$

$d\theta \to 0$, $\cos d\theta \to 1$

So, $\boxed{d\theta = \frac{1}{Nm \cot\theta}}$

it gives the angular half width of m th principal maximum, So,

$$1/Nm = \frac{md\lambda}{(a+b)\sin\theta}$$

by solving we get resolving power of grating as

$$R.P. \Rightarrow \boxed{\frac{\lambda}{d\lambda} = Nm}$$

$\frac{\lambda}{d\lambda}$ increases if N increased.

Now by substitute the value of 'm'

$$\frac{\lambda}{d\lambda} = \frac{N(a+b)\sin\theta}{\lambda} = \frac{W\sin\theta}{\lambda}$$

Where, $W = N(a+b)$ is the total width of the ruled surface of the grating. So a change in N for a given W, will not change the R.P.

But, the angular dispersive power of the grating increases with the increase in no. of rullings per unit length, i.e. $\frac{1}{a+b}$

- Thus a grating with higher dispersive power does not necessarily posses a higher resolving power.

putting $\theta = 90°$, we get,

$$\left(\frac{\lambda}{d\lambda}\right)_{max} = \frac{W}{\lambda}$$

Now for the oblique incident it can be easily shown that,

$$\boxed{\left(\frac{\lambda}{d\lambda}\right)_{max} = 2\times\frac{W}{\lambda}}$$

- R.P. of a Prism Spectrometer ⇒ $\boxed{\frac{\lambda}{d\lambda} = t\frac{dn}{d\lambda}}$

- Reference Book –

A Text book on light – B Ghosh & K.G. Majumdar