Analog Modulation

~~Subpos.~~

Need for frequency translation:—

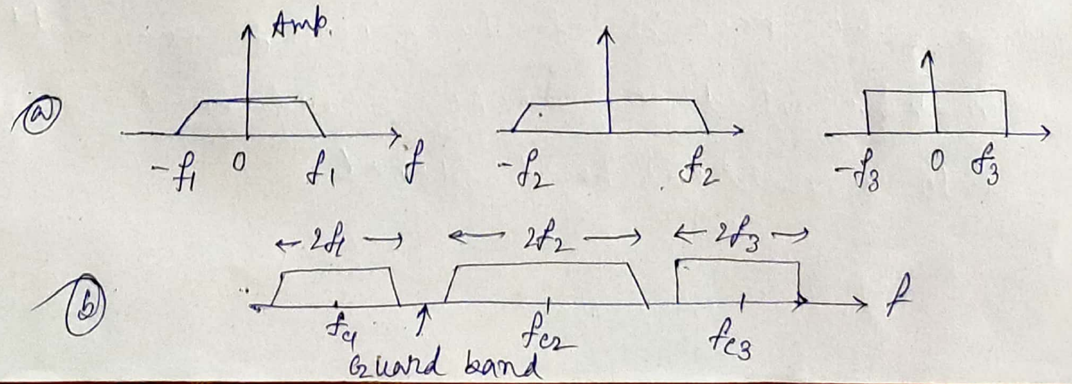
Suppose a signal is band limited so that the frequency range extending from f_1 to f_2. The process of frequency translation is one in which the original signal is replaced with a new signal whose spectral range extends from f_1' to f_2' and the information ~~is~~ of the original signal can be recovered from new one.

There are a number of purposes —

1) frequency multiplexing —

Suppose we have several different signals in the same spectral range. All these signals are to be transmitted along same communication channel in such a manner that they can be recovered seperately.

Suppose one signal is translated to frequency range f_1' to f_2', another signal to frequency range f_1'' to f_2'' and so on. If these new frequency ranges do not overlap, the signals maybe seperated at receiver end by appropriate band pass filter.



(a) → original spectrum

(b) → spectrum after translation.

② practicability of antenas:—

when free space is used as communication channel, antennas radiate and receive signal. Antennas operate effectively only when their dimensions are of the order of magnitude of wavelength of the signal. An audio signal of frequency 1 KHz has wavelength 300 km which is ~~a~~ impractical length of antennas. The required length is to be reduced by translating the signal to higher frequency.

③ narrowbanding:—

Assuming the audio range from 50 Hz to 10^4 Hz, the ratio of the heighest to lowest wavelength is 200. So, an antenna suitable for use at one end of the range would be entirely too short or too long for the other end. Suppose the spectrum were translated so that it occupied the range (10^6+50) Hz to (10^6+10^4) Hz. The ratio of heighest to lowest wavelength would be 1.01.

So, the process of frequency translation maybe used to change a wideband signal into a narrow band signal.

Fourier series expansion and its use :-

A period function of time $v(t)$ having a fundamental period T_0 can be represented as Fourier Series —

$$v(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos \frac{2\pi n t}{T_0} + \sum_{n=1}^{\infty} B_n \sin \frac{2\pi n t}{T_0}$$

A_0 is the average value of $v(t)$ given by

$$A_0 = \frac{1}{T_0} \int_{-T_0/2}^{+T_0/2} v(t)\, dt$$

A_n and B_n are given by

$$A_n = \frac{2}{T_0} \int_{-T_0/2}^{+T_0/2} v(t) \cos \frac{2\pi n t}{T_0}\, dt$$

and $$B_n = \frac{2}{T_0} \int_{-T_0/2}^{+T_0/2} v(t) \sin \frac{2\pi n t}{T_0}\, dt$$

Alternative form of Fourier series is

$$v(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos\left(\frac{2\pi n t}{T_0} - \phi_n\right)$$

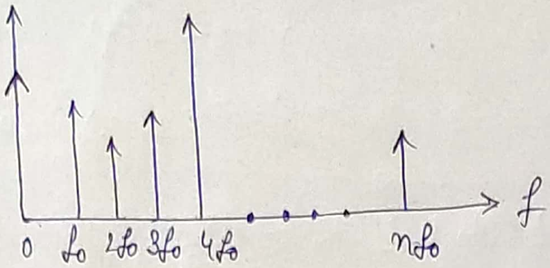
where, $C_0 = A_0$

$$C_n = \sqrt{A_n^2 + B_n^2}$$

$$\phi_n = \tan^{-1} \frac{B_n}{A_n}$$

The Fourier series is seen to consist of a summation of harmonics of a fundamental frequency $f_0 = \frac{1}{T_0}$. The coefficients C_n are called spectral amplitudes. C_n is the spectral amplitude of component $C_n \cos(2\pi n f_0 t - \phi_n)$ at frequency $n f_0$.

A typical amplitude spectrum is shown below –



A vertical line has been drawn having a length equal to the spectral amplitudes. Such amplitude spectrum lack the phase information.

The information carried by ϕ_n if plotted against f will give phase spectrum.

<u>Exponential form of fourier Series –</u>

The exponential form of Fourier series has extensive applications in communication theory. This form is given by

$$v(t) = \sum_{n=-\infty}^{+\infty} V_n e^{j2\pi n t/T_0}$$

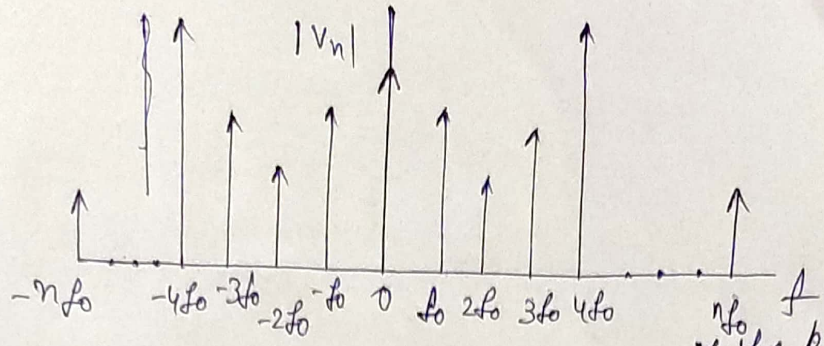
$$\text{Where, } V_n = \frac{1}{T_0} \int_{-T_0/2}^{+T_0/2} v(t)\, e^{-j2\pi n t/T_0}\, dt$$

$$\text{Here, } V_n = V_{-n}^{*}$$

$$V_0 = C_0$$

$$\text{and } V_n = \frac{C_n}{2} e^{-j\phi_n}$$

V_n's are the spectral amplitude of the spectral components $V_n e^{j2\pi n f_0 t}$. The amplitude spectrum of V_n's shown below ~~is the~~ corresponds to the amplitude spectrum of C_n's.



Here, $V_0 = C_0$. Each spectral line ~~here~~ of the previous sp. is replaced by 2 spectral lines here, each of half amplitude one at freq. f and one at freq. $-f$.

The previous one is called a single sided spectrum while the later is called two-sided spectrum. It is more convenient to use two sided spectrum.

We can also write amplitude spectrum of periodic signal $v(t)$ as

$$V(f) = \sum_{-\infty}^{+\infty} V_n \, \delta(f - nf_0) \text{; where, } f_0 = \frac{1}{T_0}.$$

$V(f)$ refers to fourier transform of $v(t)$.

The Fourier Transform:—

A periodic waveform may be expressed as a sum of spectral components. These components have finite amplitudes and are seperated by finite frequency intervals $f_0 = \frac{1}{T_0}$. Now suppose we increase ~~with~~ the timeperiod T_0 without limit. So the pulse around $t=0$ remains in place but all other pulses move outward from $t=0$ as $T_0 \to \infty$

As $T_0 \to \infty$, the spacing between spectral components become infinitesimal. The spectral amplitudes also become infinitesimal.

The Fourier series of the periodic waveform

$$v(t) = \sum_{n=-\infty}^{+\infty} V_n e^{j2\pi n f_0 t}$$

becomes

$$v(t) = \int_{-\infty}^{+\infty} V(f)\, e^{j2\pi f t}\, df$$

The finite spectral amplitudes V_n are analogous to the infinitesimal spectral amplitudes $V(f)\,df$. $V(f)$ is called amplitude spectral density or generally Fourier transform of $v(t)$.

The Fourier transform is given by

$$V(f) = \int_{-\infty}^{+\infty} v(t)\, e^{-j2\pi f t}\, dt$$

in correspondence with V_n, which is given by

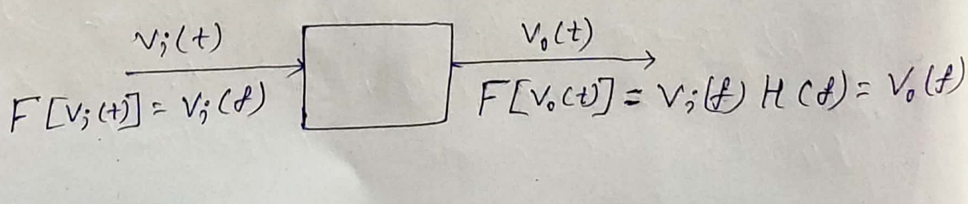
$$V_n = \frac{1}{T_0} \int_{-T_0/2}^{+T_0/2} v(t)\, e^{-j2\pi n f_0 t}\, dt$$

Let, $H(f)$ be the transfer function of a network. If the input signal is $v_i(t)$, output signal will be

$$v_o(t) = \int_{-\infty}^{+\infty} H(f)\, V(f)\, e^{j2\pi f t}\, df$$

$$\therefore F[v_o(t)] = H(f)\, F[v_i(t)]$$

or $$V_o(f) = H(f)\, V_i(f)$$



Amklitude Modulation

In amklitude modulation, the amklitude of a high frequency wave called carrier wave is varied in accordance with the instantaneous value of a low frequency message signal called modulating signal.

Let the carrier be rekresented by

$$v_c = A_c \cos w_c t \quad ; \quad w_c = 2\pi f_c$$

and the modulating signal modulating signal is given by

$$v_m = A_m \cos w_m t . \quad ; \quad w_m = 2\pi f_m$$

where, $f_c >> f_m$

An amklitude modulated signal can be exkressed as -

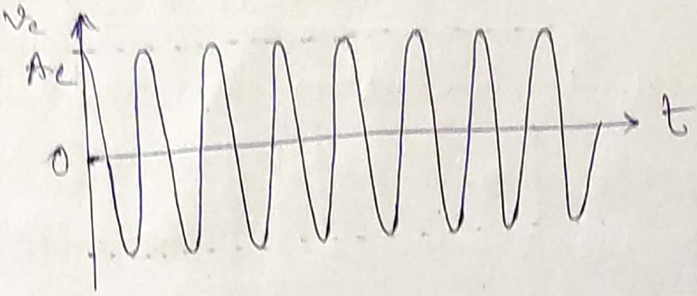
$$v(t) = (A_c + K A_m \cos w_m t) \cos w_c t$$

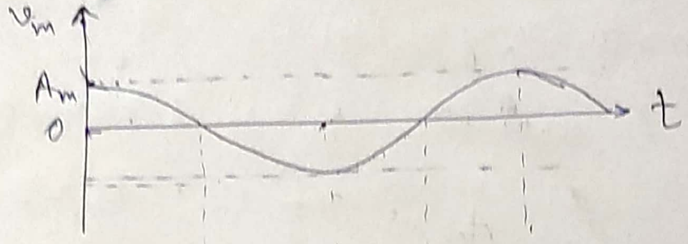
$$= A_c \left(1 + \frac{K A_m}{A_c} \cos w_m t\right) \cos w_c t$$

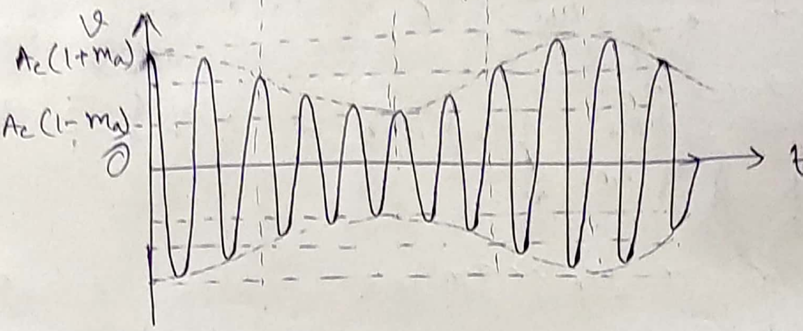
$$= A_c (1 + m_a \cos w_m t) \cos w_c t$$

Where, $m_a = \frac{K A_m}{A_c}$ is called modulation index or degree of modulation.

The tykical waveforms of $v_m(t)$, $v_c(t)$ and $v(t)$ are shown in figure.







The maximum and minimum amplitudes of AM wave are $A_{max} = A_c(1+m_a)$ and $A_{min} = A_c(1-m_a)$

So, the modulation index

$$m_a = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

A wave is said to be fully modulated if $m_a = 1$. Measuring A_{max} and A_{min} it is possible to find modulation index m_a.

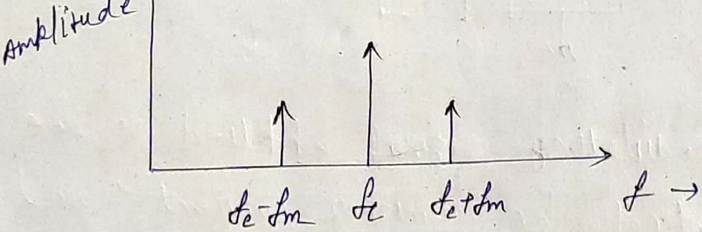
Frequency spectrum:—

$$v(t) = A_c(1 + m_A \cos \omega_m t) \cos \omega_c t$$

$$= A_c \cos \omega_c t + m_A A_c \cos \omega_m t \cdot \cos \omega_c t$$

$$= A_c \cos \omega_c t + \frac{m_A A_c}{2} \cos(\omega_c + \omega_m)t + \frac{m_A A_c}{2} \cos(\omega_c - \omega_m)t$$

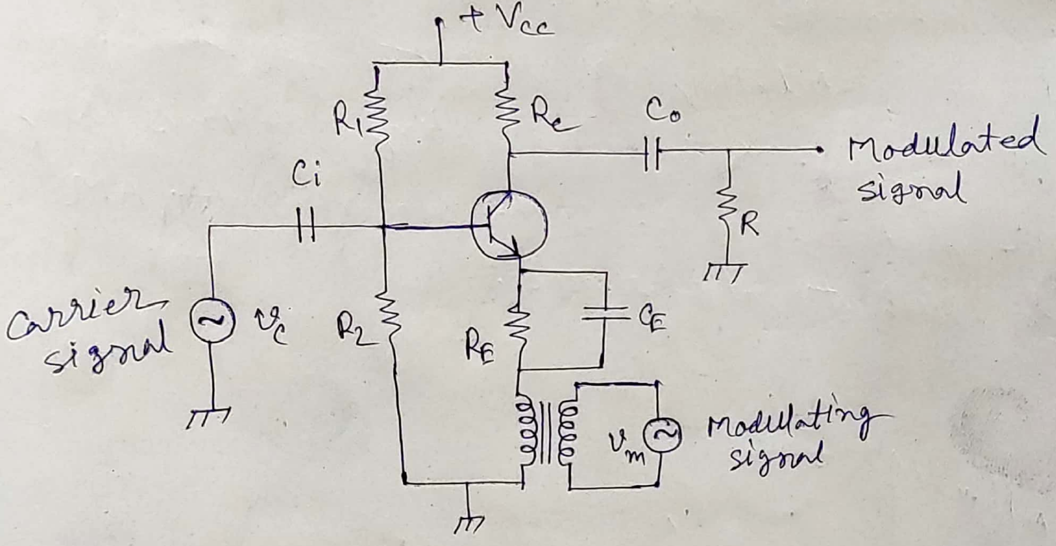
This shows that an AM wave Consists of the Carrier frequency f_c and two other frequencies $f_c \pm f_m$, Called Sideband Comkonents. The amklitude of Sideband Comkonent are equal. The Comkonent with freq. f_c+f_m is Called uprer Sideband and f_c-f_m is called lower Sideband. These Sidebands Contains full information while the carrier contains no information, but helps in transmission process. For full transmission of the AM wave, the bandwidth equal to $(f_c+f_m)-(f_c-f_m) = 2f_m$. The modulating voltage may be complex with a number of frequency comkonents. Each Comkonent produces two sidebands. All such Sidebands will lie within the bandwidth equal to twice the maximum modulating frequency. If the channel band width be 20KHZ, broadcasting stations using carrier Seperation of 20KHz Can transmit AM waves without interference.



Generation of AM :-

Various methods are available for the generation of AM waves. There are two basic types – linear modulators and squarelaw modulators. Normal broadcasting uses linear modulators whereas squarelaw modulators are suitable for low voltage applications such as mobile communications.

As an example, a BJT emitter modulator circuit is shown below –



The modulating signal applied to the emitter control the emitter current and hence controls the gain of the amplifier and hence the amplifier output becomes an AM waveform. R_1, R_2, R_c, R_E and V_{cc} establish the d.c operating point.

As the modulating signal is applied to the ~~Collector~~ emitter, it causes emitter current to vary about the dc value.

$$i_e = I_E + K_1 A_m \cos \omega_m t$$

K_1 is a proportionality constant.

So, collector current at that instant

$$i_c = \alpha i_e = \alpha (I_E + K_1 A_m \cos \omega_m t)$$

$$= I_c + K_2 A_m \cos \omega_m t$$

Voltage gain of the amplifier is controlled by i_c. Hence;

$$A_v = K_3 i_c = K_3 (I_c + K_2 A_m \cos \omega_m t)$$

K_3 is a proportionality constant.

Since, the input carrier voltage is

$$v_i = A_c \cos \omega_c t$$

So, the output of the amplifier is

$$v_o = A_v v_i$$

$$= K_3 A_c (I_c + K_2 A_m \cos \omega_m t) \cos \omega_c t$$

$$= A (1 + m_a \cos \omega_m t) \cos \omega_c t$$

where, $A = K_3 A_c I_c$ and $m_a = \frac{K_2 A_m}{I_c}$

So, the amplifier output is an AM wave.

Method of frequency translation:-

A signal may be translated to a new spectral range by multiplying the signal with an auxiliary sinusoidal signal.

Let a signal be

$$V_m(t) = A_m \cos \omega_m t = A_m \cos 2\pi f_m t$$

$$= \frac{A_m}{2}\left(e^{j2\pi f_m t} + e^{-j2\pi f_m t}\right)$$

~~Am is the mo~~

The auxiliary signal be

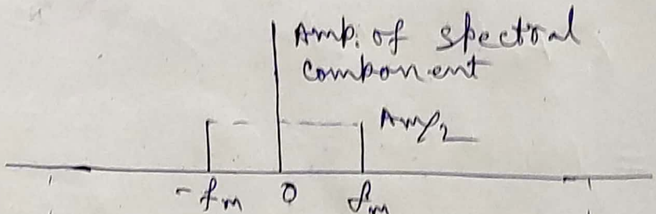
$$V_c(t) = A_c \cos \omega_c t$$

$$= \frac{A_c}{2}\left(e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right)$$

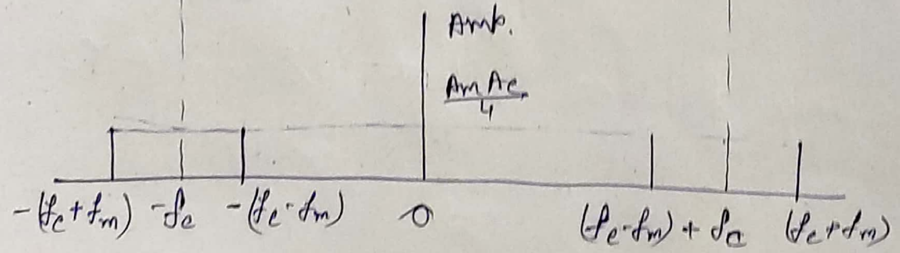
A_m is the modulating ~~a~~ amplitude and A_c is the carrier amplitude.

$$V_m(t)\, V_c(t) = A_m A_c \cos \omega_m t \cos \omega_c t$$

$$= \frac{A_m A_c}{4}\left(e^{j(\omega_c+\omega_m)t} + e^{-j(\omega_c+\omega_m)t} + e^{j(\omega_c-\omega_m)t} + e^{-j(\omega_c-\omega_m)t}\right)$$

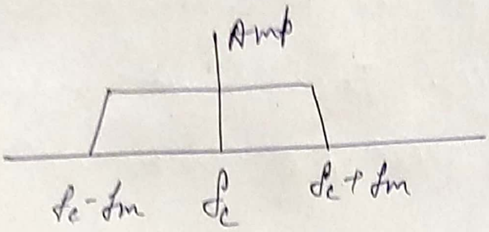


Sp. of $V_m(t)$

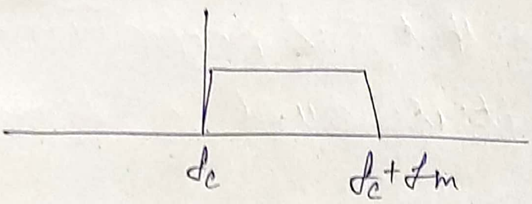


Sp. of $V_m(t)\, V_c(t)$

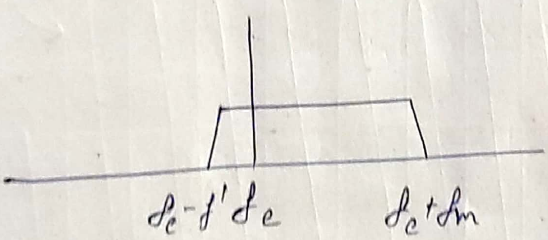
There exists two side bands around the carrier signal. If the carrier is transmitted with the side bands, it is called ~~DSC~~ DSB-C signal. If only one sideband is transmitted it is called SSB-SC signal. If a part of one sideband ~~and~~ with carrier and other side band is transmitted, it is called VSB signal.



DSB-C signal



SSB signal



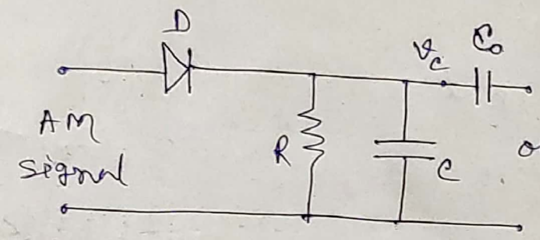
VSB signal

~~DSB-C modulator~~

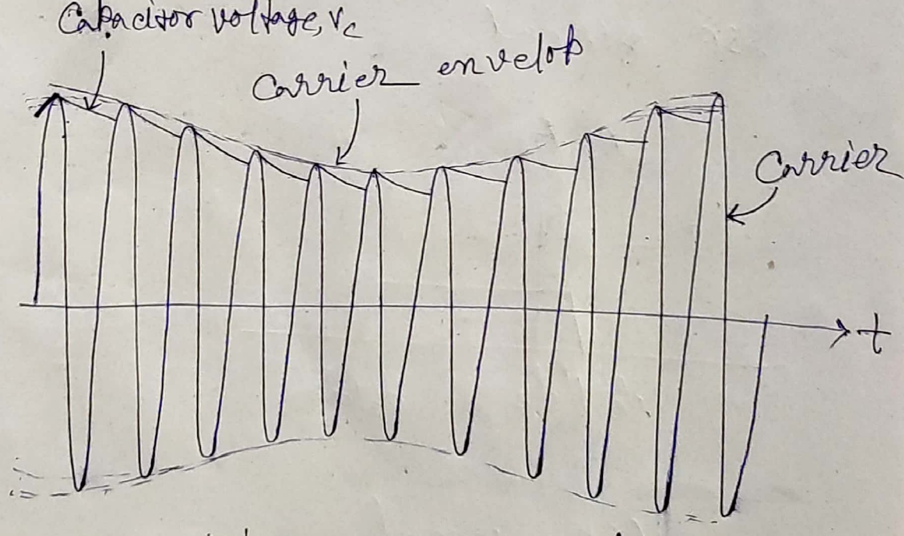
~~DSB-C modulator can be generated using a mixer (multiplier)~~

~~Amplitude~~
AM wave demodulation:-

Demodulation is the process of extracting original message signal from the modulated wave. The simplest circuit to recover message from AM signal is Envelop detector or a diode rectifier. The circuit consists of of a diode and a RC combination.



Since its output is proportional to the input voltage, it is called a linear detector.



~~Let us initially assume that the input is of fixed~~

Input wave form and output voltage v_c

Let us initially assume that the input is of fixed amplitude and R is ~~not~~ present. In this case, the capacitor charges to the peak +ve voltage of the carrier. The capacitor holds this peak voltage and the diode ~~doesn't~~ would not conduct. The carrier amplitude is increased and the diode again conducts and the capacitor charges to new peak. To allow the capacitor voltage to follow the carrier peaks when the carrier amplitude is decreasing, it is necessary to include the resistor R to discharge the capacitor. In this case the capacitor voltage v_c has the form shown. The capacitor charges to the peak and decays slightly between the ~~carriers~~ cycles. The value of RC is so chosen that ~~the~~ v_c decreases at least to the decrease in carrier amplitude.

In practice the ~~ca~~ cycle is so fast that the envelope is greatly exaggerated. If RC ~~the~~ time constant is very large, ~~carrier~~ v_c falls ~~sto~~ at a slower rate than the modulation envelope. The negative peaks of the modulating wave would be clipped off. It is nown as diagonal clipping. If RC = 0, the output falls at ~~a~~ a larger rate.

Let the envelope of the modulated wave be represented by

$$v(t) = A_c (1 + m_a \cos \omega_m t)$$

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Let the envelope of the modulated wave be represented by

$$v(t) = A_c \left(1 + m_a \cos \omega_m t\right)$$

At time t_0,

$$V_0 = v(t_0) = A_c(1 + m_a \cos \omega_m t_0)$$

Its slope is

$$\left.\frac{dv}{dt}\right|_{t=t_0} = -A_c m_a \omega_m \sin \omega_m t_0$$

Let, the capacitor be charged to the envelope voltage at $t = t_0$ and starts discharging.

So, v_c at $t > t_0$ is

$$v_c = V_0 e^{-(t-t_0)/RC}$$

So, rate of change of voltage across C at $t = t_0$ is

$$\left.\frac{dv_c}{d(t-t_0)}\right|_{t=t_0} = -\frac{v_0}{RC} = -\frac{A_c(1+m_a \cos \omega_m t_0)}{RC}$$

The slope of the curve of capacitor voltage at $t=0$ must be smaller than the slope of the envelope of the modulated wave.

So,

$$-\frac{A_c}{RC}(1 + m_a \cos \omega_m t_0) \ngtr -A_c m_a \omega_m \sin \omega_m t_0$$

$$\text{or } RC \leq \frac{1}{\omega_m\left[\dfrac{m_a \sin \omega_m t_0}{1+m_a \cos \omega_m t_0}\right]}$$

For RC to be minimum, $\dfrac{m_a \sin \omega_m t_0}{1+m_a \cos \omega_m t_0}$ must be maximum and this is the worst situation.

$$\therefore \frac{d}{dt_0}\left[\frac{m_a \sin \omega_m t_0}{1+m_a \cos \omega_m t_0}\right]=0$$

$$\Rightarrow \frac{\omega_m \cos \omega_m t_0}{1+m_a \cos \omega_m t_0}+\frac{m_a \omega_m \sin^2 \omega_m t_0}{(1+m_a \cos \omega_m t_0)^2}=0$$

$$\Rightarrow (1+m_a \cos \omega_m t_0)\cdot \omega_m \cos \omega_m t_0 + m_a \omega_m \sin^2 \omega_m t_0 = 0$$

$$\Rightarrow \omega_m \cos \omega_m t_0 + m_a \omega_m = 0$$

$$\Rightarrow \cos \omega_m t_0 = -m_a$$

and $\sin \omega_m t_0 = \sqrt{1-m_a^2}$

So, $\cancel{Rc}$ $\boxed{RC \lesssim \frac{1}{\omega_m}\cdot\frac{\sqrt{1-m_a^2}}{m_a}}$

When the modulation index is ~~s~~small

$$\sqrt{1-m_a^2} \approx 1$$

$$\therefore RC \lesssim \frac{1}{\omega_m \cdot m_a}$$

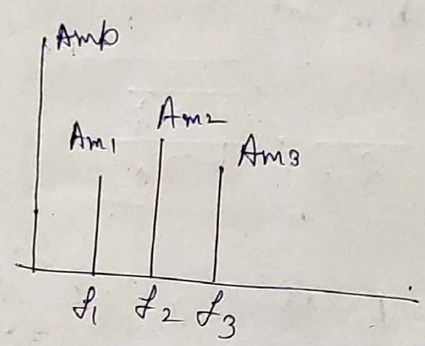
The diode envelope detector is widely used. It introduces less distortion particularly ~~a~~for larger percentage modulation. It can handel comparatively large input signal. Its main disadvantage is that it requires appreciable input power.

Spectrum and Power efficiency

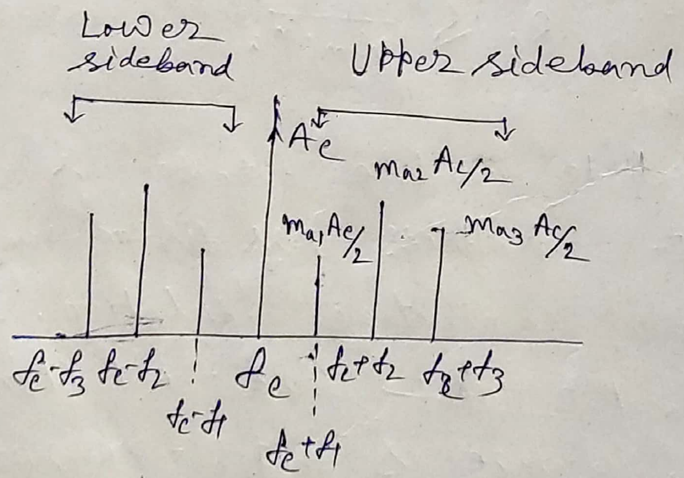
The spectrum of a AM signal is similar to the spectrum of a signal which results from multiplication except that in the former case the carrier of frequency f_c is present.

If the modulating signal contains three frequencies given by, $v_m(t) = A_{m1} \cos \omega_1 t + A_{m2} \cos \omega_2 t + A_{m3} \cos \omega_3 t$,

one sided spectrum of this baseband signal is shown below -



The spectrum of modulated carrier is shown below -



The basic eqn of AM modulation is

$$v(t) = A_c [1 + m(t)] \cos \omega_c t$$

$m(t)$ is the modulating signal.

$\therefore\ v(t) = A_c \cos w_c t + A_c m(t) \cos w_c t$.

The 1st term represents that power ~~to~~ needed to transmit the carrier is $P_c = A_c^2/2$ and 2nd term represents power transmitted in sidebands, P_s (say). Carrier power doesnot carry any information but helps in demodulation.

So, power efficiency in AM transmission is

$$\eta = \frac{P_s}{P_c + P_s}$$

If $m(t) = \cancel{A} m_a \cos w_m t$, [$m_a$ = modulation index]

$$v(t) = A_c \cos w_c t + \frac{A_c \cancel{A} m_a}{2} \left[\cos(w_c + w_m)t + \cos(w_c - w_m)t \right]$$

$$\therefore P_s = \frac{1}{2}\left[\frac{(A_c \cancel{A} m_a)^2}{2} + \frac{(A_c m_a)^2}{2} \right]$$

$$= \frac{(A_c m_a)^2}{2}$$

$$\therefore \eta = \frac{m_a^2}{2 + m_a^2}$$

If $m_a = 1$, $\eta = \frac{1}{3}$ which is maximum.

Single Side Band modulation (SSB) :-

The baseband signal may be recovered from a DSB-SC signal by multiplying a second time with the same carrier. Baseband signal can be recovered in a similar manner if only one side band is available. Suppose a baseband signal is multiplied by a carrier ~~w_ct~~ $\cos w_c t$, giving rise to two sidebands at $w_c + w$ and $w_c - w$. Let us consider we have filtered out one side band and left with upper side band $w_c + w$. If, this side band signal is multiplied by ~~original~~ $\cos w_c t$ we get,

$$y(t) = \cos(w_c + w)t \cdot \cos w_c t$$

$$= \frac{1}{2}\left[\cos(2w_c + w)t + \cos wt\right]$$

So, we can generate a signal at $(2w_c + w)$ and original base band spectral component at w. Since, we can recover message signal from only one side band, it is economic to do so. Two single - side band (SSB) can now be accomodated within the spectral range previously occupied by a DSB-SC system.

The baseband signal may not be recovered ~~from~~ by the use of a diode demodulator. In this case, the single side~~baseband~~ signal will consists of a single sinusoid of freq. $f_c + f$ and there is no amplitude variation which the diode modulator can respond.

SSB modulation by phase shift :-

Assuming the upper and lower side band components to be $\cos(\omega_c+\omega)t$ and $\cos(\omega_c-\omega)t$. the two side bands can be represented as

$$m_{SSB-L}(t) = \cos(\omega_c - \omega_m)t$$

$$= \cos\omega_c t \cdot \cos\omega_m t + \sin\omega_c t \cdot \sin\omega_m t$$

and

$$m_{SSB-H}(t) = \cos(\omega_c + \omega_m)t$$

$$= \cos\omega_c t \cdot \cos\omega_m t - \sin\omega_c t \cdot \sin\omega_m t$$

The carrier signals of angular frequency ω_c which are applied to the modulators differ in phase by 90°. Similarly the baseband signals before applying to the modulators is passed through a 90° phase shifting network ~~both~~ so that there is a 90° phase shift between any spectral component of the baseband signal applied to one modulator and the like frequency component applied to the other modulator.

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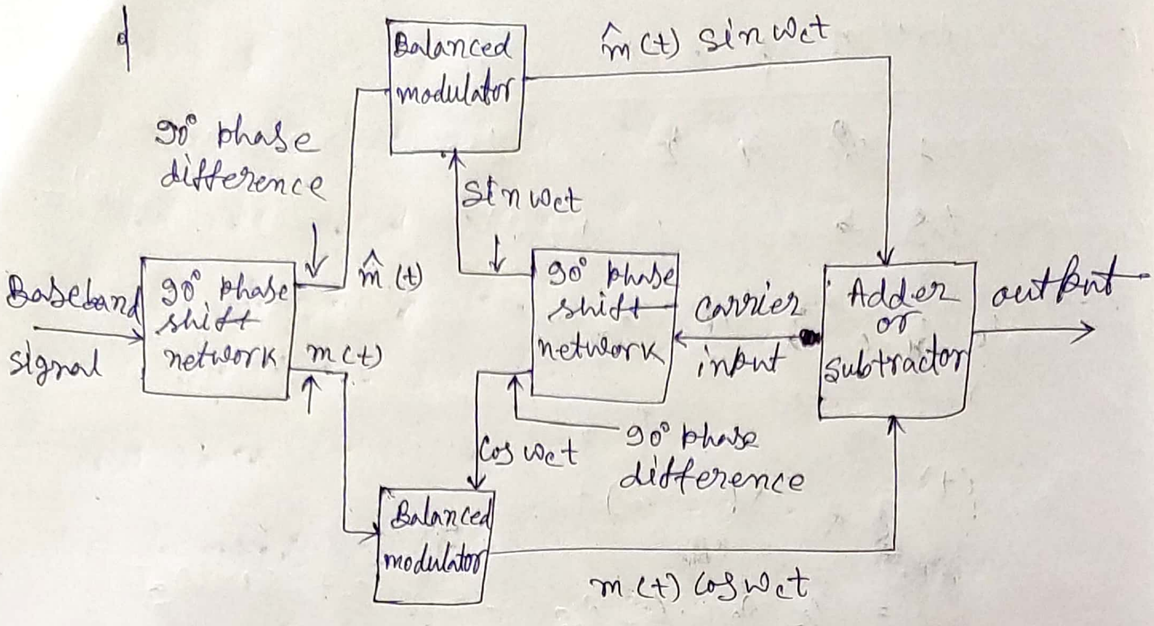


Method of generating SSB signal using balanced modulators and phase shifters.

Let us assume that the baseband signal is sinusoidal and appears at the input of one modulator as $\cos \omega_m t$ and as $\sin \omega_m t$ in the other. Let the carrier be $\cos \omega_c t$ at one modulator and $\sin \omega_c t$ at the other.

Then the outputs of the balanced modulators (multipliers) are

$$\cos \omega_m t \cos \omega_c t = \frac{1}{2}\left[\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t\right]$$

and $\sin \omega_m t \sin \omega_c t = \frac{1}{2}\left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\right]$

If this waveforms are added, the lower sideband results and if subtracted, upper sideband appears.

In general,

$$m(t) = \sum_{i=1}^{m} A_i \cos(\omega_i t + \theta_i)$$

The output of the SSB modulator is in general

$$m(t) \cos \omega_c t \pm \hat{m}(t) \sin \omega_c t$$

where, $\hat{m}(t) = \sum_{i=1}^{m} A_i \sin(\omega_i t + \theta_i)$

~~It is less~~

This method is less popular than filter method. The reason is the need of precise 90° shift of phase shifter over a large freq. range, each modulator carefully balanced etc.

SSB Demodulator —

Coherent detector

Baseband recovery is achieved by heterodyning the received signal with a local carrier signal which is synchronous (coherent) with the carrier used at transmitting end to generate the sideband.

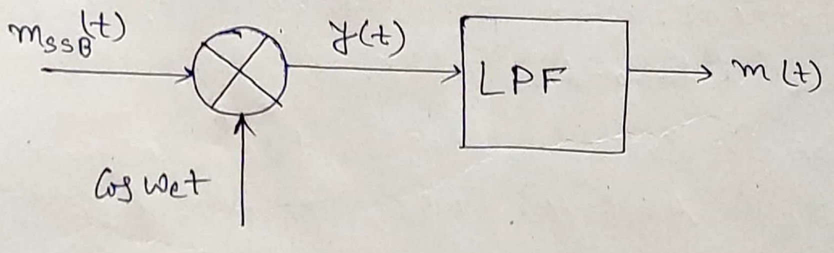
Suppose the carrier received is $\cos \omega_c t$ and the local carrier is $\cos(\omega_c t + \theta)$. Then in DSB-SC the spectral component $\cos \omega t$ will, upon demodulation appear as $\cos \omega t \cos \theta$. In SSB, the spectral component will appear in the form $\cos(\omega t - \theta)$. In one case a phase offset in carrier affects the amplitude of the recovered signal and for $\theta = \pi/2$, may result in a total loss of the signal.

In other case the offset produces a phase change but not in amplitude change. Humans are less sensitive in phase distortion. So, SSB can be used for speech ~~translati~~ transmission even if the local carrier has phase offset.

Let the local oscillator carrier has an angular frequency offset $\Delta\omega$ and so be of form $\cos(\omega_c + \Delta\omega)t$. In DSB-SC the recovered signal will have form $\cos\omega_c t \cos\Delta\omega t$, but in SSB the form is $\cos\cos(\omega_c + \Delta\omega)t$. In one case the signal reappears with a 'warble', i.e., an amplitude fluctuation ~~with~~ at the rate $\Delta\omega$. In other case the amplitude remains fixed.

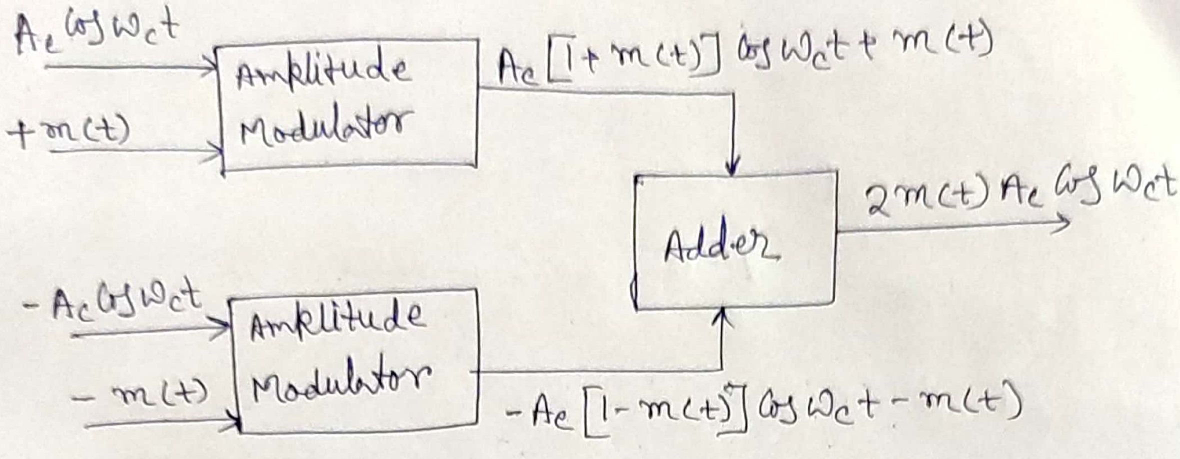
When used for synchronization, the carrier is called 'pilot carrier'.

The coherent detection scheme for SSB modulated signal is shown below -



$y(t)$ when passed through a LPF of band width that of $m(t)$ supresses components around $2\omega_c t$ and recovers ~~the~~ ~~ist~~ ~~to~~ $m(t)$, the message signal.

Balanced Modulator



Angle Modulation

In angle modulation, the spectral components in the modulated waveform depend on the amplitude as well as the frequency of the spectral components in the baseband signal. The modulation signal is not linear and superposition doesnot apply. Such a signal has the form

$$v(t) = A_c \cos\left[\omega_c t + \phi(t)\right]$$

The phase angle $\phi(t)$ is a function of the base band signal.

$v(t)$ can be written as

$$v(t) = A_c \cos[\omega_c t + \phi(t)] = Re\left(A_c e^{j(\omega_c t + \phi(t))}\right)$$

~~The instantaneous angular frequency is~~

$$\cancel{\omega = \frac{d}{dt}\left[\omega_c t + \phi(t)\right]}$$

We may consider that the angle $\omega_c t + \phi(t)$ of $v(t)$ undergoes a modulation around the angle $\theta = \omega_c t$. The waveform $v(t)$ is therefore a representation of a signal which is modulated in phase.

The instantaneous angular frequency is

$$\omega = \frac{d}{dt}\left[\omega_c t + \phi(t)\right]$$

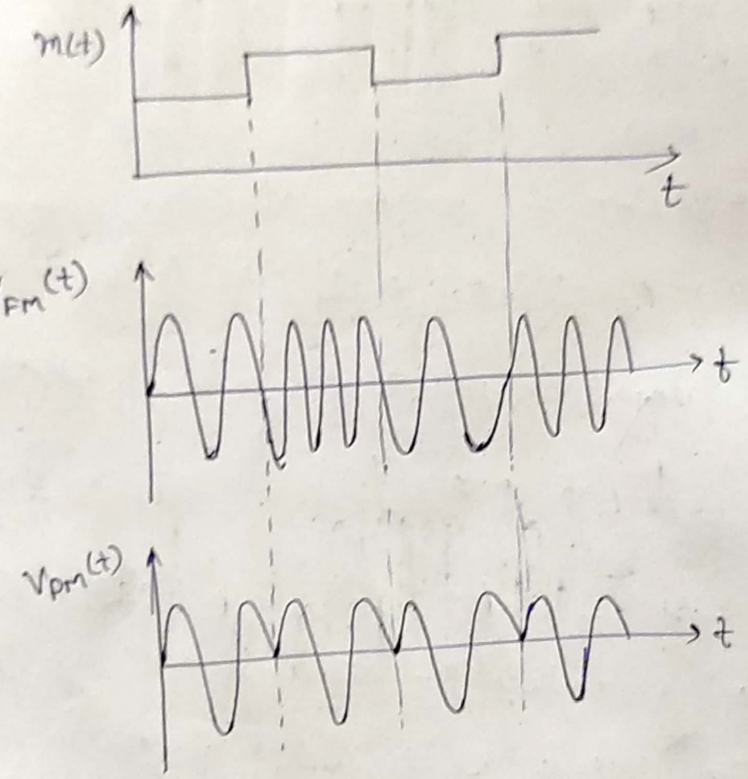
$$\text{or } f = \frac{1}{2\pi}\frac{d}{dt}\left[\omega_c t + \phi(t)\right] = \frac{\omega_c}{2\pi} + \frac{1}{2\pi}\frac{d}{dt}\phi(t)$$

So, the waveform is therefore modulated in frequency.

If the frequency variation about the nominal frequency ω_c is small, i.e., if $\frac{d\phi}{dt}$

if $\frac{d\phi(t)}{dt} << \omega_0$, the resultant waveform will have an appearance readily recognizable as a 'sine wave' albeit with a period which changes from cycle to cycle.

For a modulating signal as square wave



$$\cancel{\frac{d\phi(t)}{dt} = 2\pi(f-f_c)}$$

We might arrange that the phase $\phi(t)$ is directly proportional to the modulating signal or a direct proportionality between the modulating and the the derivative $\frac{d\phi(t)}{dt}$.

$$\frac{d\phi(t)}{dt} = 2\pi(f-f_c).$$

We refer the 1st type as phase modulation or PM and the 2nd type as frequency modulation or FM.

Frequency modulation —

Let the carrier be represented by

$$v_c = A_c \cos(\omega_c t + \theta)$$

$\phi(t) = \omega_c t + \theta$ is the instantaneous phase.

Let the modulating signal be-

$$v_m = A_m \cos \omega_m t$$

Instantaneous angular frequency of the FM wave may be written as

$$\omega(t) = \omega_c + K A_m \cos \omega_m t$$

k is proportionality constant.

Instantaneous frequency $f = \frac{\omega}{2\pi}$ varies between $f_{max} = f_c + \frac{K A_m}{2\pi}$ and $f_{min} = f_c - \frac{K A_m}{2\pi}$ about the value f_c at rate of f_m cycles per second.

So, the freq. deviation

$$\Delta f = f_{max} - f_c = f_c - f_{min} = \frac{K A_m}{2\pi}$$

Δf is proportional to A_m but independent of f_m. Instantaneous phase angle is given by

$$\phi(t) = \int_0^t \omega(t)\, dt$$

$$= \int_0^t (\omega_c + K A_m \cos \omega_m t)\, dt$$

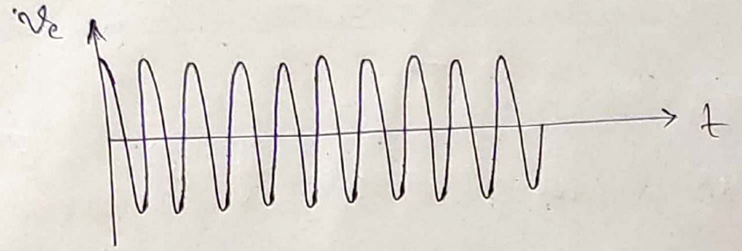
$$= \omega_c t + \frac{K A_m}{\omega_m} \sin \omega_m t$$

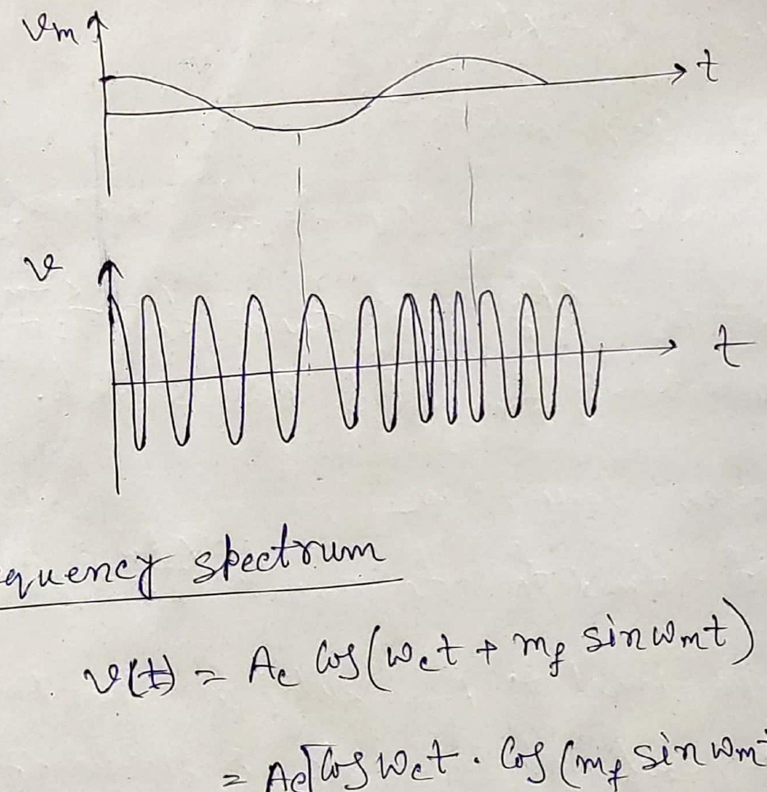
$$= \omega_c t + m_f \sin \omega_m t$$

Where, $m_f = \frac{KA_m}{\omega_m} = \frac{\Delta f}{f_m}$ is called the modulation index. It is the ratio of frequency deviation to the modulation frequency.

The frequency modulated wave is represented by

$$v = A_c \cos(\omega_c t + m_f \sin \omega_m t)$$





Frequency spectrum

$$v(t) = A_c \cos(\omega_c t + m_f \sin \omega_m t)$$

$$= A_c[\cos \omega_c t \cdot \cos(m_f \sin \omega_m t) - \sin \omega_c t \cdot \sin(m_f \sin \omega_m t)]$$

$\cos(m_f \sin \omega_m t)$ is an even periodic function having an angular frequency ω_m.

Since, the function is even,

$$\cos(m_f \sin \omega_m t) = J_0(m_f) + 2\sum_{n=1}^{\infty} J_{2n}(m_f) \cos 2n\omega_m t$$

and for being odd the function $\sin(m_f \sin \omega_m t)$ can be written as

$$\sin(m_f \sin \omega_m t) = 2\sum_{n=0}^{\infty} J_{2n+1}(m_f) \sin(2n+1)\omega_m t$$

where, $J_n(m_f)$ is the Bessel function of order n.

So, we can write,

$$v(t) = A_c\Big[J_0(m_f)\cos \omega_c t + \sum_{n=1}^{\infty} J_{2n}(m_f)\{\cos(\omega_c + 2n\omega_m)t$$

$$+ \cos(\omega_c - 2n\omega_m)t\}$$

$$- \sum_{n=0}^{\infty} J_{2n+1}(m_f)\{\cos(\omega_c - \overline{2n+1}\,\omega_m)t$$

$$- \cos(\omega_c + \overline{2n+1}\,\omega_m)t\}\Big]$$

This eqn. shows that there are infinite no. of sidebands in FM wave.

For small value of m_f

$$J_0(m_f) \simeq 1 - \left(\frac{m_f}{2}\right)^2$$

and $J_n(m_f) \simeq \frac{1}{n!}\left(\frac{m_f}{2}\right)^n$

For m_f very small, the FM signal consists of a carrier and a single pair of side bands with frequency $\omega_c \pm \omega_m$. This type of FM signal is called narrowband FM signal.

It has been found by numerical computation that for undistorted detection of FM, receiver must have a bandwidth equal to

$$2(\Delta f + f_m) = 2(m_f + 1)f_m$$

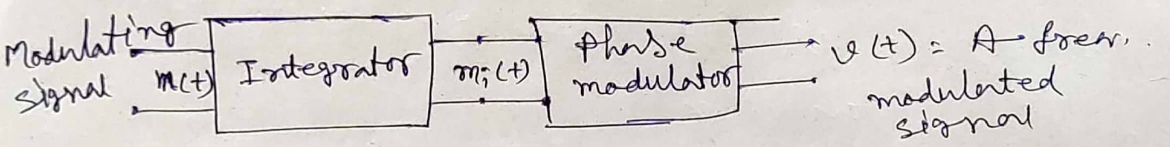
f_m is the maximum modulating frequency.
This is known as Carson's rule of thumb.

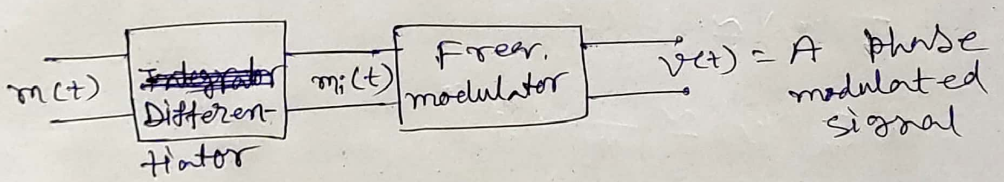
FM broadcasting stations are allotted a channel width of 200 KHz for each. Within the same geographical region, allotted frequencies are seperated by atleast 200 KHz in the frequency band 88—108 MHz.

Relationship between FM and PM:—

Consider the following diagrams in which the phase modulator block represents a device which furnishes an output $v(t)$ which is phase modulated by the input signal $m_i(t)$

So, $v(t) = A_c \cos[w_c t + K_p m_i(t)]$





Let the ~~s~~ waveform be derived as the integral of modulating signal.

So, $m_i(t) = K' \int_{-\infty}^{t} m(t)\, dt$

with $K_f = K' K_p$,

$v(t) = A_c \cos\left[w_c t + K_f \int_{-\infty}^{+\infty} m(t)\, dt\right]$

The instantaneous angular frequency is

$$\omega = \frac{d}{dt}\left[\omega_c t + k_f \int_{-\infty}^{t} m(t)\,dt\right] = \omega_c + k\,m(t)$$

The deviation of the instantaneous freq. from carrier frequency $\frac{\omega_c}{2\pi}$ is

$$\Delta f = f - f_c = \frac{k}{2\pi}\, m(t)$$

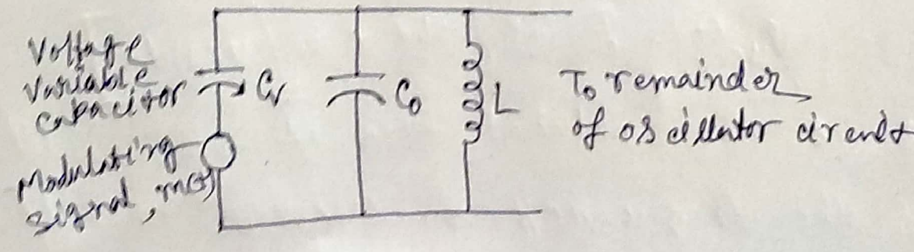
Since, the deviation is directly proportional to the modulating signal, the combination of integrator and a phase modulator constitutes a device for producing a FM output. Similarly the combination of the differentiator and a frequency modulator generates a phase-modulated output.

FM generation by parameter variation:—

The generator which produces the carrier of an FM waveform is a tuned oscillator. These have very large frequency determined by the resonant frequency of an inductance-capacitance combination. The frequency of oscillation is

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Such an LC combination, a parallel combination here is shown in fig below



The capacitor consists here a fixed capacitor C_0 which is shunted by a voltage variable capacitor C_v, called a varicap is one whose capacitance depends on the dc biasing voltage maintained across its electrodes.

Semiconductor diodes when operated with a reverse bias acts as one.

In the circuit the modulating signal varies voltage across cr and as a consequence value of cr changes which causes the change in oscillator frequency. Any oscillator whose frequency is controlled by modulating signal voltage is called a voltage-controlled oscillator or VCO.

Frequency modulation may be achieved by the variation of any element or parameter on which the frequency depends. A capacitor, resistor or inductor whose value can be varied with an electrical signal. Reverse biased junctions may serve as voltage variable capacitors. FETS can act as variable resistors. The inductance of magnetic cored inductor called a saturable reactor can be varied by changing dc biasing current through the winding.

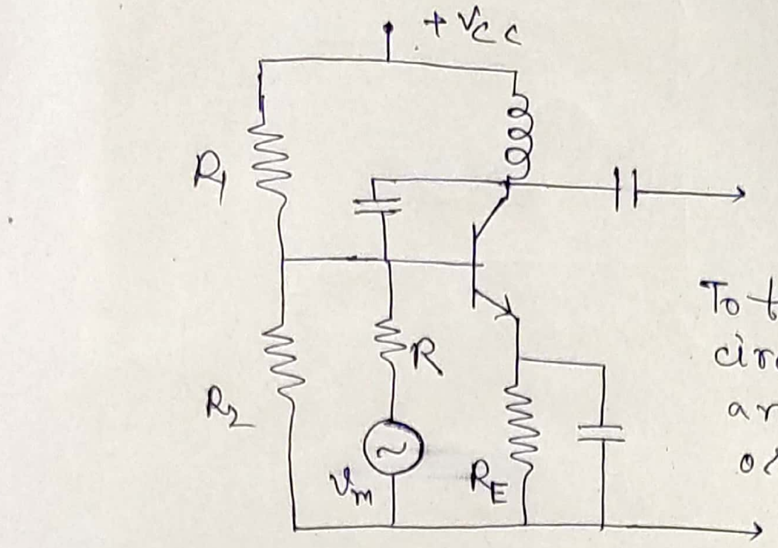
The block level relation between variable voltage $v(t)$ as input and $v_{osc}(t)$ as output is given by

$$v_{osc}(t) = B \cos\left[w_c t + G_0 \int_{-\infty}^{t} v(\lambda)\,d\lambda\right]$$

w_c is the natural freq of the oscillator, G_0 is the frequency sensitivity in rad/volt

Instantaneous angular frequency is

$$W_i(t) = W_c + G_0\, v(t)$$

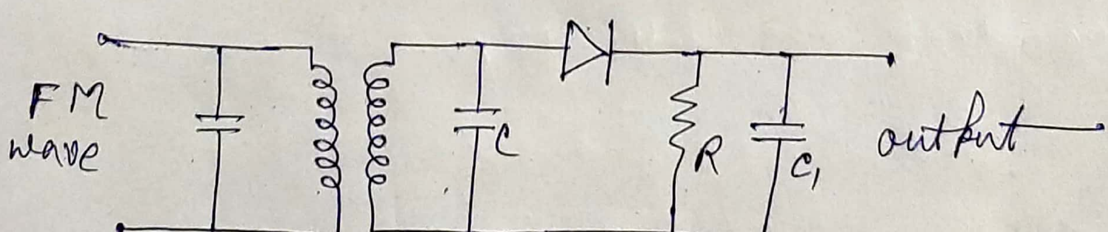


BJT reactance modulator for FM generation

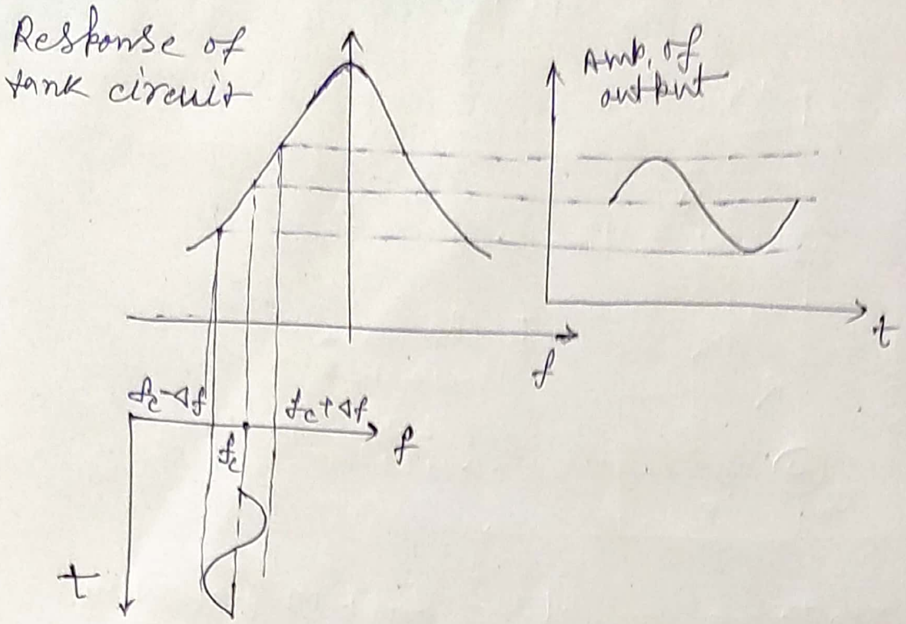
Detection of FM wave :—

FM detector also called discriminator is used to extract the original signal. Usually the detection takes place in two steps. First the frequency variation of FM is converted into amplitude variation. The voltage with this amplitude variation is then applied to a diode detector to extract modulating signal. There are different types of FM detector.

A slope detector circuit is shown below —



The graphical analysis is shown below.



The figure shows how the circuit converts the frequency variation of FM into corresponding amplitude variation. The frequency of FM varies between $f_c - \Delta f$ and $f_c + \Delta f$ with a frequency equal to modulating frequency f_m. So, the tuned circuit output is amplitude modulated with frequency f_m. This AM wave is then detected by the diode detector.

Because of nonlinearity, in the response curve, the output voltage becomes distorted for FM with large frequency deviation.

~~PM Modulator and~~

Spectrum of frequency modulated signal:—

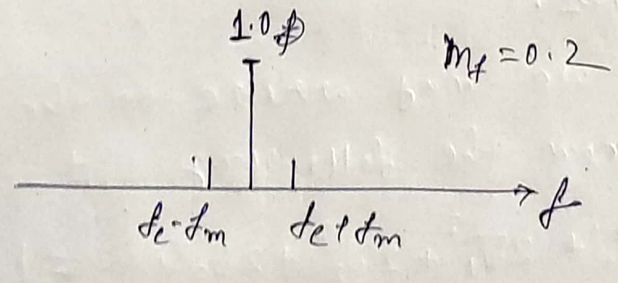
Frequency modulated signal is represented as

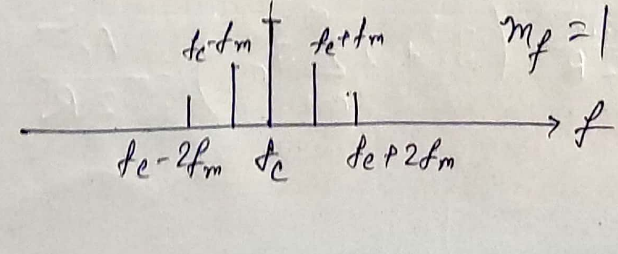
$$v(t) = A_c \cos(\omega_c t + m_f \sin \omega_m t)$$

$$= A_c\Big[J_0(m_f)\cos\omega_c t + \sum_{n=1}^{\infty} J_{2n}(m_f)\{\cos(\omega_c + 2n\omega_m)t + \cos(\omega_c - 2n\omega_m)t\} - \sum_{n=0}^{\infty} J_{2n+1}(m_f)\{\cos(\omega_c - \overline{2n+1}\,\omega_m)t - \cos(\omega_c + \overline{2n+1}\,\omega_m)t\}\Big]$$

$$= A_c\Big[J_0(m_f)\cos\omega_c t - J_1(m_f)\{\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\}$$
$$+ J_2(m_f)\{\cos(\omega_c - 2\omega_m)t + \cos(\omega_c + 2\omega_m)t\}$$
$$- J_3(m_f)\{\cos(\omega_c - 3\omega_m)t - \cos(\omega_c + 3\omega_m)t\}$$
$$+ \cdots\Big]$$

The spectrum is composed of a carrier with an amplitude $A_c J_0(m_f)$ and a set of sidebands spaced symmetrically on either side of the carrier at frequency seperations of f_m, $2f_m$, $3f_m$ etc. The spectrum is unlike AM where only two sidebands appear.





Power of FM Signal :-

The FM signal is given by

$$v(t) = A_c \cos(\omega_c t + m_f \sin \omega_m t)$$

So, the average power is given by

$$P_v = \frac{A_c^2}{2R}$$ and is independent of m_f.

When the carrier is modulated to generate FM signal, the power in the sidebands may appear only at the expense of the power originally in the carrier.

Again, we may arrive to the same conclusion from the following that

$$J_0^2 + 2J_1^2 + 2J_2^2 + 2J_3^2 + \dots = 1$$

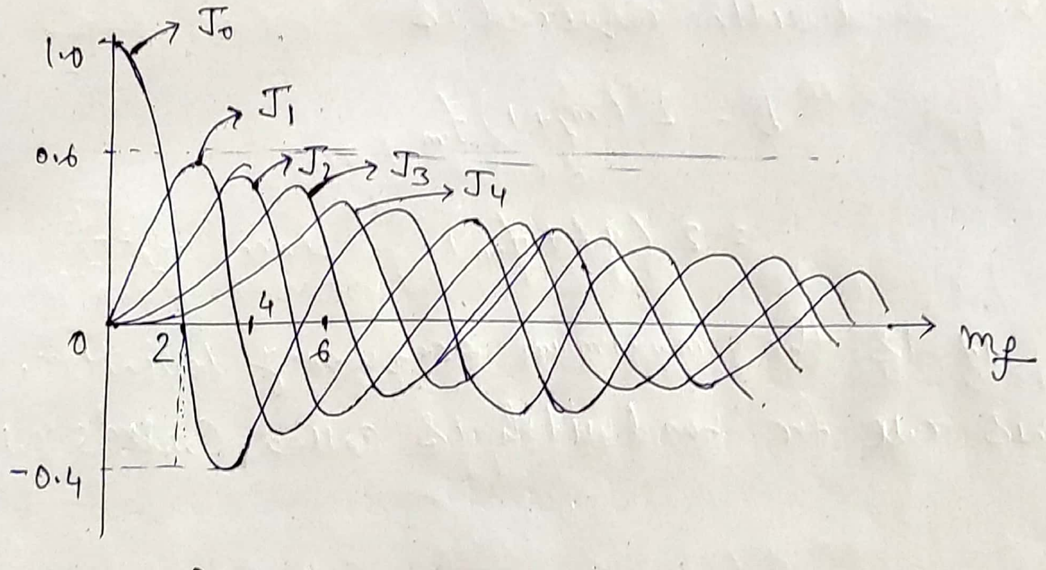
$$\therefore P_v = \frac{A_c^2}{2R}\left(J_0^2 + 2\sum_{n=1}^{\infty} J_n^2\right) = \frac{A_c^2}{2R}$$

At various values of m_f, $J_0(m_f) = 0$. At these values of m_f all the power is in the sidebands.

Band width of FM signal —

When an FM signal is modulated, the number of sidebands is infinite and the bandwidth required to encompass such a signal is similarly infinite in extent. In practice, for any value of m_f, so large a fraction of total power is confined to the sidebands which lie within some finite bandwidth that no serious distortion of the signal results if the sidebands outside this bandwidth are lost.

~~Note that~~;



Note that except for $J_0(m_f)$, each $J_n(m_f)$ hugs the zero axis initially and as n increases, the corresponding J_n remains very close to zero axis upto a large value of m_f. It is found experimentally that the distortion resulting from bandlimiting an FM signal is tolerable as long as 98% or more of the power is passed by the bandlimiting filter.

Let us consider, $m_f = 1$.

The power contained in the terms $n = 0, 1$ and 2 is

$$P = \frac{A_c^2}{2R}\left[J_0^2(1) + 2J_1^2(1) + 2J_2^2(1)\right]$$

$$= \frac{A_c^2}{R} \cancel{\times 0} \cdot \left[0.289 + 0.193 + 0.013\right]$$

$$= \frac{A_c^2}{R} \times 0.495$$

$$= P_t \times 99\%$$

We note that the value of n for 98% power transmission always occur after $n = m_f + 1$. Thus for sinusoidal modulation, the bandwidth required is

$$B = 2(m_f + 1) f_m$$

$$= 2(\Delta f + f_m) \quad \text{since, } m_f = \frac{\Delta f}{f_m}$$

Δf is the maximum frequency deviation. This rule for bandwidth is called Carson's rule.

Spectrum of 'Constant Bandwidth' FM –

Let the modulating signal voltage

$$v_m = A_m \cos 2\pi f_m t$$

In a phase modulating system the phase angle $\phi(t)$ would be proportional to this modulating signal so that

$$\phi(t) = K' A_m \cos 2\pi f_m t \quad \text{with } K' \text{ a Constant.}$$

The phase deviation is $m_f = K' A_m$ and for Constant A_m, the bandwidth occupied increases linearly with modulating frequency since $B \cong 2 m_f f_m$ $= 2K' A_m f_m$.

Let $\phi(t) = \dfrac{K}{2\pi f_m} A_m \sin 2\pi f_m t$. For this case $m_f = \dfrac{K A_m}{2\pi f_m}$ and the bandwidth is $B \cong \dfrac{2K}{2\pi} A_m$ independent of f_m. In this case the instantaneous frequency is

$$f = f_c + \frac{K}{2\pi} A_m \cos 2\pi f_m t$$

Since, the instantaneous frequency is proportional to the modulating signal, the initially phase modulated signal is now frequency modulated signal. The bandwidth is $B \cong 2\Delta f$.

PM Modulator and Demodulator:—

We have seen the relation between frequency and phase modulation. If we have a frequency modulator, we can use that for phase modulation too by sending the modulating signal first through a differentiator and then to FM modulator.

A similar argument follows for PM demodulators too. A frequency discriminator generates an output $y(t)$ which can be approximated in our linear region of interest as

$y(t) \propto m(t)$, where $m(t)$ is message signal.

If $\theta(t)$ is phase of the FM system then $\frac{d\theta(t)}{dt} \propto m(t)$. Thus

$$y(t) \propto \frac{d\theta(t)}{dt}$$

For a phase modulated input signal, $\theta(t) \propto m(t)$.

Thus discriminator output

$$y(t) \propto \frac{d\,m(t)}{dt}$$

$$\Rightarrow y(t) = K\frac{d\theta(t)}{dt}$$; K is a proportionality constant.

By placing an integrator after frequency discriminators in FM demodulation we can get phase demodulator that recovers message signal from a phase modulated input.