Study Material on

$\boxed{\text{Statistical Hypothesis}}$

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Statistical Hypothesis

* **Statistical Hypothesis:** Any statement or assertion about a statistical population or the values of its parameters is called a Statistical Hypothesis. There are two types of hypothesis —Simple and Composite. V.H –'06,'05,'99,'95 '07,

* **Simple Hypothesis:** A statistical hypothesis which specifies the population completely (i.e., the probability distribution and all parameters are known) is called a Simple Hypothesis. V.H –'05,'98

Example1: Consider a normal(m,σ) population. The hypothesis $H_0: m=2, \sigma=0.1$ is a simple hypothesis. The point set consists of the single point $(2,0.1)$ in the parametric plane.

* **Composite Hypothesis:** A statistical hypothesis which does not specify the population completely (i.e., either the form of probability distribution or some parameters remain unknown) is called a Composite Hypothesis. V.H –'05,'98

Example2: Consider a normal(m,σ) population. The hypothesis $H_0: m=2$ is a composite hypothesis. Since σ is unspecified, the point set consists of the straight line $m=2$.

* **Test of Hypothesis (or test of significance):** A test of hypothesis is a procedure which specifies a set of "rules for decision" whether to 'accept' or 'reject' the hypothesis under consideration. V.H –'06

* **Null Hypothesis:** A statistical hypothesis which is set up (i.e., assumed) and whose validity is tested for possible rejection on the basis of sample observations is called a Null Hypothesis. It is denoted by H_0 and tested against alternatives. Tests of hypotheses deal with rejection or acceptance of null hypothesis only. V.H –'05,'01,'94

* **Alternative hypothesis:** A statistical hypothesis which differs from the null hypothesis is called an Alternative Hypothesis, and is denoted by H_1. The alternative hypothesis is not tested, but its acceptance (rejection) depends on the rejection (acceptance) of the null hypothesis. Alternative hypothesis contradicts the null hypothesis. The choice of an appropriate critical region depends on the type of hypothesis, viz. whether both-sided, one-sided (right/left) or specified alternative. V.H – '01

* **Test statistic:** A function of sample observations(i.e., statistic) whose computed value determines the final decision regarding acceptance or rejection of H_0, is called a Test Statistic.

* **Critical Region:** The set of values of the test statistic which lead to rejection of the null hypothesis is called Critical Region of the test. The probability with which a true null hypothesis is rejected by the test is often referred to as "Size" of the Critical Region. V.H –'96

Geometrically, a sample $x_1, x_2, \ldots\ldots, x_n$ of size n is looked upon as just a point x, called Sample point, within the region of all possible samples, called the Sample Space(W). The critical region is then defined as a subset(w) of those sample points which lead to the rejection of the null hypothesis.

V.H-106/103

Level of significance: The maximum probability with which a true null hypothesis is rejected is known as Level of Significance of the test, and is denoted by α.

In framing decision rules, the level of significance is arbitrarily chosen in advance depending on the consequence of statistical decision.

Generally, 5% or 1% level of significance is taken, although other levels such as 2% of $\frac{1}{2}$% is also used.

V.H-103 106 189 187 194 107

Type I Error (or Error of First Kind): This is the error committed in rejecting a null hypothesis by the test when it is really true. The critical region is so determined that that the probability of Type I error does not exceed the level of significance of the test.

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Type II Error (or Error of Second Kind): This is the error committed in accepting a null hypothesis by the test when it is really false. The probability of Type II error depends on the specified value of the alternative hypothesis, and is used to evaluating the efficiency of a test.

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Power of the test: The probability of rejecting a false null hypothesis is called Power of the test. Therefore, Power is the probability of drawing a correct conclusion by the test, when the null hypothesis is false. For a specified value of the parameter consistent with the alternative hypothesis,

Power=1 – Probability of Type II error.

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Best critical region: A particular critical region which would minimize the probability of committing type II error i.e., maximize the power of the test. This critical region, if existent, will be called the best critical region and the corresponding test the best test or the most powerful test at the given significance level.

Steps in test of significance:

(1) Set up the "Null Hypothesis" H_0 and the "Alternative Hypothesis" H_1 on the basis of the given problem. The null hypothesis usually specifies the values of some parameters involved in the population: $H_0(\theta = \theta_0)$. The alternative hypothesis may be any one of the following types: $H_1(\theta \neq \theta_0)$, $H_1(\theta > \theta_0)$, $H_1(\theta < \theta_0)$. The type of alternative hypothesis determines whether to use a two-tailed or one-tailed test (right or left tail).

(2) State the appropriate "test statistic" T and also its sampling distribution, when the null hypothesis is true. In large sample tests the statistic $z = (T - \theta_0)/S.E.(T)$, which approximately follows Standard Normal Distribution, is often used. In small sample tests, the population is assumed

to be Normal and various test statistics are used which follows Standard Normal, Chi-square, t or F distribution exactly.

(3) Select the "level of significance" α of the test, if it is not specified in the given problem. This represents the maximum probability of committing s Type I error, i.e., of making a wrong decision by the test procedure when in fact the null hypothesis is true. Usually, a 5% or 1% level of significance is used (If nothing is mentioned, use 5% level.)

(4) Find the "critical region" of the test at the chosen level of significance. This represents the set of values of the test statistic which lead to rejection of the null hypothesis. The critical region always appears in one or both tails of the distribution, depending on whether the alternative hypothesis is one-sided or both-sided. The area in the tails (called 'size of the critical region') must be equal to the level of significance α. For a one-tailed test, α appears in one tail and for a two-tailed test $\frac{\alpha}{2}$ appears in each tail of the distribution. The critical region is

$$T \geq T_{\alpha/2} \text{ or } T \leq T_{1-\alpha/2} \quad \text{when } H_1(\theta \neq \theta_0)$$
$$T \geq T_{\alpha} \quad \text{when } H_1(\theta > \theta_0)$$
$$T \leq T_{1-\alpha} \quad \text{when } H_1(\theta < \theta_0)$$

where T_α is the value of T such that the area to its right is α.

(5) Compute the value of the test statistic T on the basis of sample data and the null hypothesis. In large sample tests, if some parameters remain unknown they should be estimated from the sample.

(6) If the computed value of test statistic T lies in the critical region, "reject H_0"; otherwise "do not reject H_0". The decision regarding rejection or otherwise of H_0 is made after a comparison of the computed value of T with the critical value (i.e., boundary value of the appropriate critical region).

(7) Write the conclusion. If H_0 is rejected, the interpretation is: "the data are not consistent with the assumption that the null hypothesis is true and hence H_0 is not tenable". If H_0 is not rejected, "the data cannot provide any evidence against the null hypothesis and hence H_0 may be accepted to be true".

Parametric space: Let $\theta_1, \theta_2, \ldots\ldots, \theta_k$ be the unknown parameters of a univariate population. Then the ordered k-tuple of real numbers $(\theta_1, \theta_2, \ldots\ldots\ldots, \theta_k)$ can be looked upon as the co-ordinates of a point in a k-dimensional space. The parametric space denoted by P_k, is defined by $P_k = \{(\theta_1, \theta_2, \ldots\ldots\ldots, \theta_k) : \theta_1, \theta_2, \ldots\ldots, \theta_k \in R\}$, where R is the set of real numbers. Then the ordered k-tuple $(\theta_1, \theta_2, \ldots\ldots\ldots, \theta_k)$ denoted by $\tilde{\theta}$ represents a point in P_k and is called a parametric point and a statistical hypothesis H which is a statement regarding the unknown parameters can be described as $H : \tilde{\theta} \in A$ where A is a specified subset of P_k.

Simple and composite hypotheses: If P_k be the parametric space corresponding to the unknown parameters $\theta_1, \theta_2, \ldots\ldots\ldots, \theta_k$ of a univariate population then a hypothesis H described by $H : \vec{\theta} \in A$ where $\tilde{\theta} = (\theta_1, \theta_2, \ldots\ldots\ldots, \theta_k)$ and A is nonempty subset of P_k, is called a simple hypothesis if A contains exactly one element otherwise H is said to be composite hypothesis.

Test of significance(Summary):

(I) Large sample tests(Approximate):

(i) **Test for a specified proportion:** $H_0 : p = p_0$

The sampling distribution of the statistic $z = \dfrac{p - p_0}{\sqrt{\dfrac{p_0 q_0}{n}}}$ is normal$(0,1)$.

(ii) **Test for equality of two proportions:** $H_0 : p_1 = p_2$

The sampling distribution of the statistic $z = \dfrac{p_1 - p_2}{\sqrt{pq\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$ is normal$(0,1)$,

where $p = \dfrac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$.

(iii) **Test for specified mean :** $H_0 : \mu = \mu_0$

The sampling distribution of the statistic $z = \dfrac{\bar{x} - \mu}{\dfrac{\sigma}{\sqrt{n}}}$ is normal$(0,1)$

and the sampling distribution of the statistic $z = \dfrac{\bar{x} - \mu}{\dfrac{S}{\sqrt{n}}}$ is normal$(0,1)$.

(iv) **Test for equality of two means :** $H_0 : \mu_1 = \mu_2$

The sampling distribution of the statistic $z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$ is normal$(0,1)$.

The sampling distribution of $z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}}$ is normal$(0,1)$.

(v) **Test for Goodness of Fit:** H_0 **:(data support theory)**

The sampling distribution of the statistic $\chi^2 = \sum \frac{(f_0 - f_e)^2}{f_e}$ is χ^2 distribution with $n-1$ degrees of freedom.

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(2) Small sample tests (Exact):

(i) Test for a specified mean for a normal population: $H_0 : \mu = \mu_0$

The sampling distribution of the statistic $z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$ is a normal$(0,1)$

And the sampling distribution of the statistic $t = \frac{\bar{x} - \mu_0}{\frac{S}{\sqrt{n-1}}}$ is t-distribution with $n-1$ degrees of freedom.

(ii) Test for the equality of two means: $H_0 : \mu_1 = \mu_2$

The sampling distribution of the statistic $z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$ is normal$(0,1)$.

And the sampling distribution of the statistic $t = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$ is t-distribution with $n_1 + n_2 - 2$ degrees of freedom, where $s^2 = \frac{n_1 S_1^2 + n_2 S_2^2}{n_1 + n_2 - 2}$

(iii) Test for equality of two means (for correlated pairs): $H_0 : \mu_x = \mu_y$

The sampling distribution of the statistic $t = \frac{\bar{d}}{\frac{S}{\sqrt{n-1}}}$ is a t-distribution with $n-1$ degrees of freedom, where $d = x - y$

(iv) Test for equality of two standard deviations (μ_1, μ_2 are unknown): $H_0 : \sigma_1 = \sigma_2$.

The sampling distribution of the statistic $F = \frac{s_1^2}{s_2^2}; (s_1 > s_2)$ is a F distribution with degrees of freedom $(n_1 - 1, n_2 - 1)$.

Note: This a right tailed test.

√ H-104 **Chi-square test of goodness of fit:** A very powerful test for testing the significance of the discrepancy between the theory and experiment was given by Prof. Karl Pearson in 1900 and is known as "Chi-square test of goodness of fit".

If $f_i (i = 1, 2, ..., n)$ is a set of observed (experimental) frequencies and $e_i (i = 1, 2, ..., n)$ is the corresponding set of expected (theoretical or hypothetical) frequencies, then Karl Pearson's chi-square, given by:

$$\chi^2 = \sum_{i=1}^{n} \left[\frac{(f_i - e_i)^2}{e_i} \right], \left(\sum_{i=1}^{n} f_i = \sum_{i=1}^{n} e_i \right)$$ follows chi-square distribution with $(n-1)$ degrees of freedom.

H-106, 199, 195 **Q1. How a statistical hypothesis is to be tested ?**

☺. Using the following steps (systematic way) we test a statistical hypothesis.

(1) Null hypothesis: Set up the null hypothesis H_0.

(2) Alternative hypothesis: Set up the alternative hypothesis to use a single-tailed (right or left) test or two-tailed test.

(3) Level of significance: Choose the appropriate level of significance (α) depending on the reliability of the estimates and permissible risk. This is to be decided before sample is drawn i.e., α is fixed in advance.

(4) Test statistic or test criterion: Compute the test statistic $z = \frac{t - E(t)}{s.e.(t)}$, under the null hypothesis.

(5) Conclusion: We compare the computed value of z in step4 with the significance value z_α at given level of significance 'α'

If $|z| < z_\alpha$ i.e., if calculated value of z (in modulus value) is less then z_α we say it is not significant. By this we mean that the difference $t - E(t)$ is just due to fluctuation of sampling and the sample data do not provided us sufficient evidence against the null hypothesis which may therefore, be accepted.

If $|z| > z_\alpha$ i.e., if the computed value of test statistic is grater than the critical significant value, then we say that it is significant and the null hypothesis is rejected at level of significance α or confidence coefficient $1 - \alpha$.

√.H-198 **Q2. Given the population density function $f(x, \theta) = \theta e^{-\theta x}, 0 \le x < \infty, \theta > 0$. The null hypothesis $H_0 : \theta = 2$ against the one sided alternative $H_1 : \theta > 2$ will** be tested on the following procedure. H_0 should be rejected if a sample point x drawn from the population is greater than or equal to 6. Find the probability of Type I error and the power of the test.

☺. Here the probability of type I error is $P(X \geq 6 | H_0) = P(X \geq 6 | \theta = 2)$ where X is the population random variable.

Now $P(X \geq 6 | \theta = 2) = \int_6^\infty 2e^{-2x} dx$

$$= \lim_{A \to \infty} \int_6^A 2e^{-2x} dx$$

$$= \lim_{A \to \infty} \left(e^{-12} - e^{-2A}\right)$$

$$= e^{-12}$$

So the required probability of Type I error is e^{-12}.

Again the probability of Type II error is equal to

$P(X < 6 | H_1) = P(X < 6 | \theta > 2)$

$$= \int_0^6 \theta e^{-\theta x} dx \text{ where } \theta > 2$$

Power $= 1 - \left(1 - e^{-6\theta}\right) = e^{-6\theta}$ where $\theta > 2$

Q3. **Let p denote the probability of getting a head when a given coin is tossed once. Suppose that the hypothesis $H_0 : p = 0.5$ is rejected in favour of $H_1 : p = 0.6$ if 10 trials result in 7 or more heads. Calculate the probabilities of Type I and Type II error.**

V. H—'06 '04 '01 '09

☺. Here the probability of Type I error is equal to $P(X \geq 7 | H_0)$ where X is the random variable denoting the number of heads in 10 independent tosses of the given coin.

Now under $H_0 : p = 0.5$, X has binomial$(10, 0.5)$ distribution.

Then under H_0 we have $P(X = x) = {}^{10}c_x (0.5)^x (1 - 0.5)^{10-x}$ for $x = 0, 1, 2, \ldots\ldots, 10$

So $P(X \geq 7 | H_0) = {}^{10}c_7 (0.5)^7 (0.5)^3 + {}^{10}c_8 (0.5)^8 (0.5)^2 + {}^{10}c_9 (0.5)^9 (0.5)^1 + {}^{10}c_{10} (0.5)^{10}$

$$= \frac{11}{64} \simeq 0.172$$

Hence the probability of Type I error is equal to $\frac{11}{64}$.

Now the probability of Type two error is equal to $P(X < 7 | H_1)$

$= 1 - P(X \geq 7 | p = 0.6)$

Again under $H_1 : p = 0.6$, X has binomial$(10, 0.6)$ distribution.

So $P(X \geq 7 | p = 0.6)$

$= {}^{10}c_7 (0.6)^7 (0.4)^3 + {}^{10}c_8 (0.6)^8 (0.4)^2 + {}^{10}c_9 (0.6)^9 (0.4)^1 + {}^{10}c_{10} (0.6)^{10}$

$$=\frac{3^7\times 1707}{5^{10}}=0.382$$

Hence the probability of Type II error is $1-0.382=0.618$.

Q4. A drug is given to 10 patients and the increments in blood pressure were recorded to be $3,6,-2,4,-3,4,6,0,0,2$. Is it reasonable to believe that the drug has no effect on the change of blood pressure? Test at 5% significance level, assuming the population to be normal. For 9 degrees of freedom $P(t>2.262)=0.025$.

☺. Here the change $d=x-y$ in the blood pressure are given i.e., x is the final blood pressure after administering the drug and y is the initial blood pressure. We are required to test whether the mean blood pressure ~~bas~~ has changed i.e., $\mu_x\neq\mu_y$.

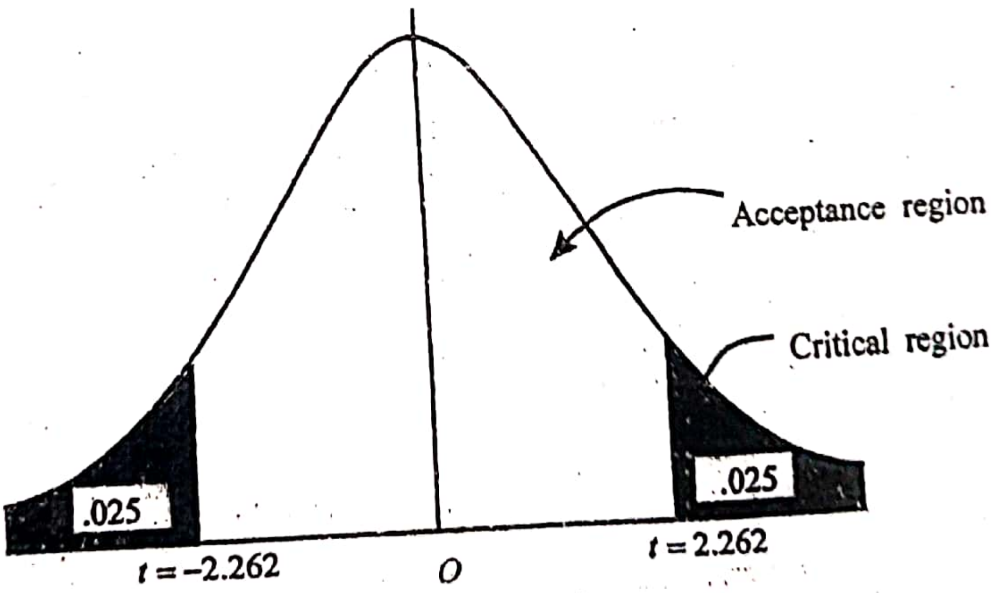
Null Hypothesis: The null hypothesis is $H_0:\mu_x=\mu_y$ i.e., mean blood pressure has not changed.

Alternative Hypothesis: The alternative hypothesis is $H_1:\mu_x\neq\mu_y$ i.e., mean blood pressure has changed after the stimulus.

Test statistic: The suitable test statistic is given by $t=\frac{\bar{d}}{\sqrt{\frac{S^2}{n-1}}}$ whose sampling distribution under H_0 is a t-distribution with $(n-1)$ degrees of freedom.

Level of significance: We are to test the null hypothesis at 5% level of significance.

Critical region: Since the alternative hypothesis is two-sided, the critical region is given by $|t|\geq t_{0.025}$ at 5% level of significance. From the given data for 9 degrees of freedom $t_{0.025}=2.262$.



Computation: for the given data, $n=10$, $\sum d_i=20$, $\sum d_i^2=130$.

$S^2=\frac{d^2}{n}-\left(\frac{d}{n}\right)^2$

Therefore , $\bar{d}=\frac{20}{10}=2$ and $S^2=\frac{130}{10}-2^2=13-4=9$

Thus $t=\frac{2}{\sqrt{\frac{9}{10-1}}}=2$

Conclusion: Since the calculated value 2 is not in $\{t:|t|\geq 2.262\}$, it is not significant. We therefore cannot reject H_0 and conclude that "the drug has no effect on the change of blood pressure".

Q5. Before an increase in excise duty on tea 400 people out of a sample of 500 were found to be tea-drinkers. After an increase in duty 400 people were tea-drinkers in a sample of 600 people. Test at 1% level of significance decrease in the consumption on tea. Given $P(U>2.33)=0.01$; U being a standard normal variate. V.H–'05

☺. We have $n_1=500$, $n_2=600$

p_1=sample proportion of tea-drinkers before increase in exercise duty=$\frac{400}{500}=0.8$

p_2=sample proportion of tea-drinkers after increase in exercise duty=$\frac{400}{600}=0.67$

Null Hypothesis: The null hypothesis is $H_0:p_1=p_2$ i.e., there is no significance difference in the consumption of tea before and after the increase in exercise duty.

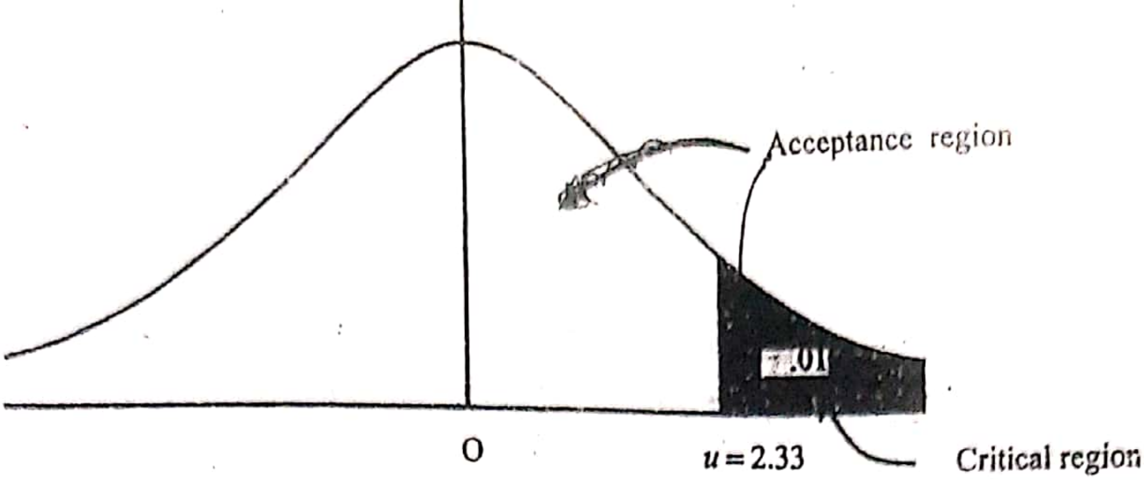
Alternative Hypothesis: The alternative hypothesis is $H_1:p_1>p_2$ i.e., there is a significant decrease in the consumption of tea after increase in the excise duty.

Test statistic: The suitable test statistic is given by $u=\frac{p_1-p_2}{\sqrt{pq\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$ whose sampling distribution under H_0 is a standard normal distribution, where

$p=\frac{n_1p_1+n_2p_2}{n_1+n_2}=\frac{400+400}{500+600}=\frac{8}{11}$ and $q=1-p=1-\frac{8}{11}=\frac{3}{11}$.

Level of significance: We are to test the null hypothesis at 1% level of significance.

Critical region: Since the alternative hypothesis is one-sided, the critical region is given by $u\geq u_{0.01}$ at 1% level of significance. From the given data $u_{0.01}=2.33$.



Computation: From the given data, $u = \dfrac{0.80-0.67}{\sqrt{\frac{8}{11}\times\frac{3}{11}\left(\frac{1}{600}+\frac{1}{500}\right)}} = 4.820529$

Conclusion: Since the calculated value 4.820529 is much greater than 2.33, it is highly significant at 1% level of significance. Hence we reject the null hypothesis H_0 and conclude that there is a significant decrease in the consumption of tea after increase in the excise duty.

Q6. The heights of 10 males of a normal population are found to be 70,67,62,67,61,68,70,64,65,66 inches. Is it reasonable to believe that the average height is greater than 64 inches? Test at 5% significance level. Assuming for 9 degrees of freedom $P(t>1.83)=0.05$.

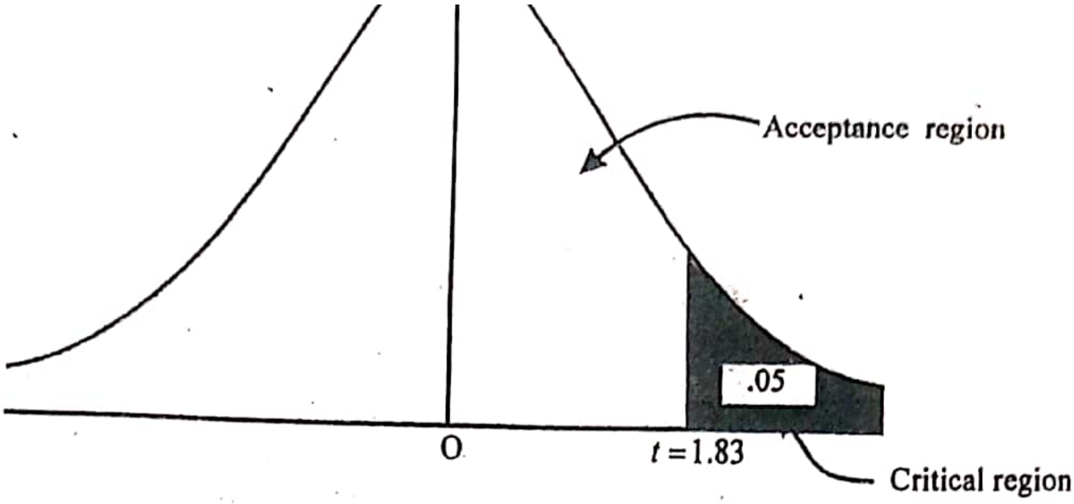
☺. Null Hypothesis: The null hypothesis is $H_0: \mu = 64$ i.e., the average height is equal to 64.

Alternative Hypothesis: The alternative hypothesis is $H_1: \mu > 64$ i.e., the average height is greater than 64.

Test statistic: The suitable test statistic is given by $t = \dfrac{\overline{x}-\mu}{\frac{s}{\sqrt{n}}}$ whose sampling distribution under H_0 is a t-distribution with $n-1$ degrees of freedom.

Level of significance: We are to test the null hypothesis at 5% level of significance.

Critical region: Since the alternative hypothesis is one-sided, the critical region is given by $t \geq t_{0.05}$ at 5% level of significance. From the given data for 9 degrees of freedom $t_{0.05} = 1.83$.



Computation: $n=10$, $\bar{x}$=sample mean=66 inches, $s=\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2}=3.055$

Therefore, $t=\frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}=\frac{66-64}{\frac{3.055}{\sqrt{10}}}=2.0702$

Conclusion: As the calculated value of t is greater than $t_{0.05}$, it is significant at 5% level of significance. Hence we reject the null hypothesis H_0 and conclude that the mean height is greater than 64 inches.

Q7. The weights(in kg) of 10 students taken at random from a college are $62, 65, 66.4, 72.1, 68.9, 76.5, 80.1, 73.4, 78.4$ and 76.3, while the population standard deviation is 5.02. Test the hypothesis at 5% significance level, that the general students have the weight greater than 70 kg.(assuming that the population of weights of the students is normal). Given $P(U>1.645)=0.05$.

☺. Here the population of weights of students is normal.

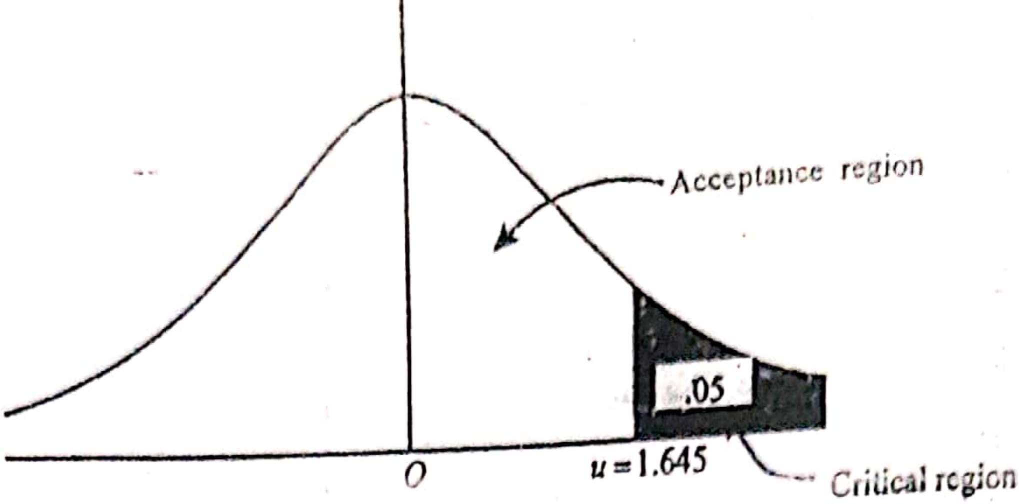
Null Hypothesis: The null hypothesis is $H_0: m=70$ i.e., the general students have the weight 70.

Alternative Hypothesis: the alternative hypothesis is $H_1: m>70$ i.e., the general students have the weight greater than 70.

Test statistic: The suitable test statistic is $u=\frac{\bar{x}-70}{\frac{\sigma}{\sqrt{n}}}$ whose sampling distribution under H_0 is normal$(0,1)$.

Level of significance: We are to test the null hypothesis at 5% level of significance.

Critical region: Since the alternative hypothesis is one-sided, the critical region is given by $u \geq u_{0.05}$ at 5% level of significance. From the given data $u_{0.05}=1.645$.



Computation: Here $n = 10$, $\sigma = 5.02$.

$$\bar{x} = \frac{62+65+66.4+72.1+68.9+76.5+80.1+73.4+78.4+76.3}{10} = 71.91$$

Therefore, $u = \dfrac{\sqrt{10}(71.91-70)}{5.02} \simeq 1.203$

Conclusion: Since the calculated value of u is not in $\{u : u \geq 1.645\}$, therefore we accept the null hypothesis H_0 and conclude the general students did not have the weight greater than 70 kg.

Q8. The wages of a factory workers are assumed to be normally distributed with mean m and variance 25. A random sample of 25 workers gives the total wages equal to 1250 units. Test the hypothesis $m = 52$ against the alternative $m = 49$ at 1% level of significance. Given that $\dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{-\infty}^{-2.32} e^{-\frac{u^2}{2}} du = 0.01$

☺. Here the population standard deviation σ is known.

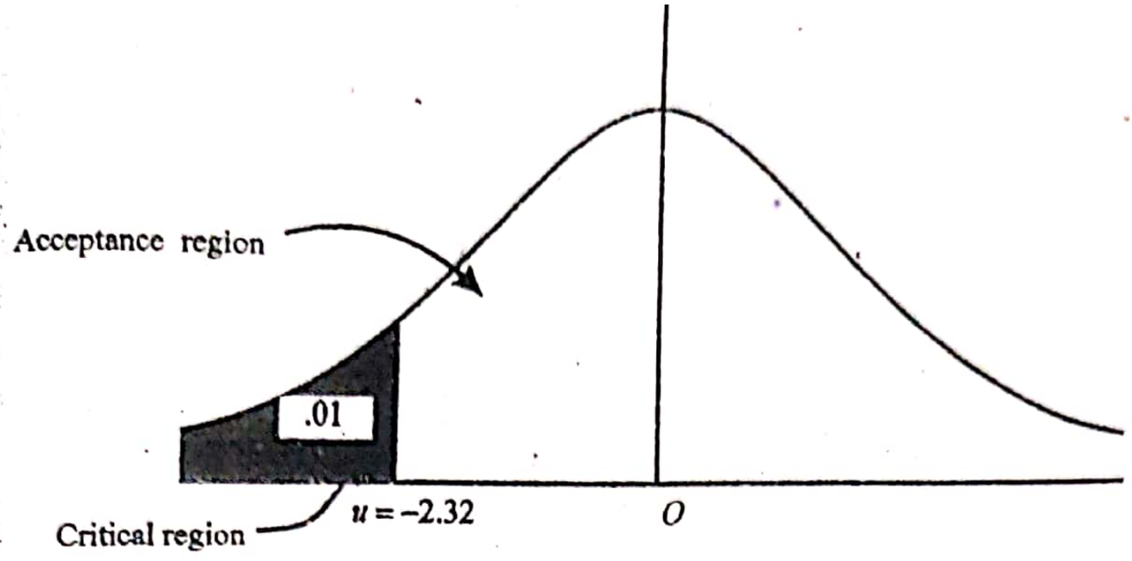
Null Hypothesis: The null hypothesis is $H_0 : m = 52$

Alternative Hypothesis: The alternative hypothesis is $H_1 : m = 49$

Test statistic: The suitable test statistic is given by $u = \dfrac{\sqrt{n}(\bar{x}-52)}{\sigma}$ whose sampling distribution is normal$(0,1)$.

Level of significance: We are to test the null hypothesis at 1% level of significance.

Critical region: Since $49 < 52$, the alternative hypothesis is left-sided and therefore, the critical region is given by $u \leq -u_{0.01}$ From the given data we have $u_{0.01} = 2.32$.



<u>Computation:</u> Here $n=25$, $\sigma=\sqrt{25}=5$

Now, $\bar{x}=\dfrac{1250}{25}=50$.

So the computed value of u is given by $u=\dfrac{\sqrt{25}(50-52)}{5}=-2$

<u>Conclusion:</u> Since the calculated value of u is not in $\{u:u\le -2.32\}$, therefore $H_0:m=52$ is accepted at 1% level of significance.

Q9. A random sample of 9 experimental animals under a certain diet give the following increase in weight:

$$\sum x_i=45\ ;\ \sum x_i^2=279$$

where $x_i\,(lbs)$ denotes the increase in weight of the animal. Assuming that the increase in weight is normally distributed as normal(μ,σ) variate, test $H_0:\mu=1$ against $H_1:\mu\ne 1$ at 5% level of significance. Given $P(t>2.306)=0.025$ for 8 degrees of freedom.

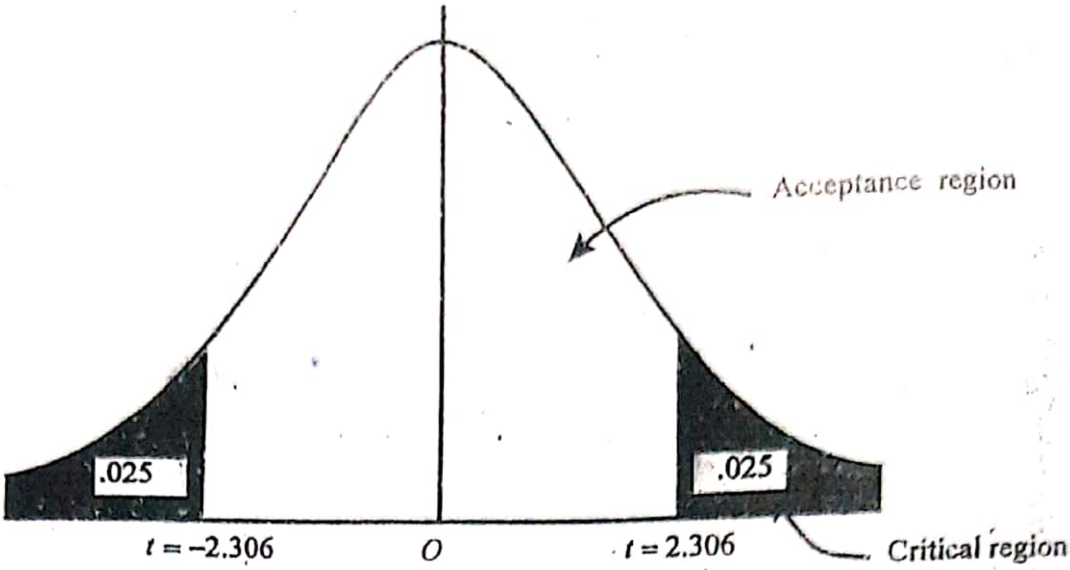
☺. <u>Null Hypothesis:</u> The null hypothesis is $H_0:\mu=1$

<u>Alternative hypothesis:</u> The alternative hypothesis is $H_1:\mu\ne 1$

<u>Test statistic:</u> The suitable test statistic is $t=\dfrac{\sqrt{n}(\bar{x}-1)}{s}$ whose sampling distribution under H_0 is a t-distribution with $n-1$ degrees of freedom.

<u>Level of significance:</u> We are to test the null hypothesis at 5% significance level.

<u>Critical region:</u> Since the alternative hypothesis two-sided, the critical region is given by $|t|\ge t_{0.025}$ at 5% significance level. From the given data $t_{0.025}=2.306$ for 8 degrees of freedom.



Computation: Here $n=9$. $\bar{x}=\frac{\sum x_i}{9}=\frac{45}{9}=5$

$S^2=\frac{1}{9}\sum x_i^2-\bar{x}^2=\frac{1}{9}\times 279-25=31-25=6$

$s^2=\frac{9}{8}\times 6=\frac{27}{4}$ or, $s=\frac{3\sqrt{3}}{2}$

Then the value of t is $\frac{2\times\sqrt{9}\times(5-1)}{3\sqrt{3}}=\frac{8}{\sqrt{3}}\simeq 4.619$

Conclusion: Since $4.619\in\{t:|t|\geq 2.306\}$, $H_0:\mu=1$ is rejected at 5% level of significance.

√Q10. Two types of chemical solutions A and B are tested for their pH values, and the following results are obtained:

	No. of sample	**Mean pH**	**Standard deviation**
Chemical A	6	7.52	0.024
Chemical B	5	7.49	0.032

Using 0.05 significance level determine whether the two types of solutions have different pH values. [Given $P(t>2.26)=0.025$, where t is a student t variate with 9 degrees of freedom.

☺. Here we assume that pH values of chemical A form a normal population with mean m_1 and the pH values of chemical B form a normal population with mean m_2.

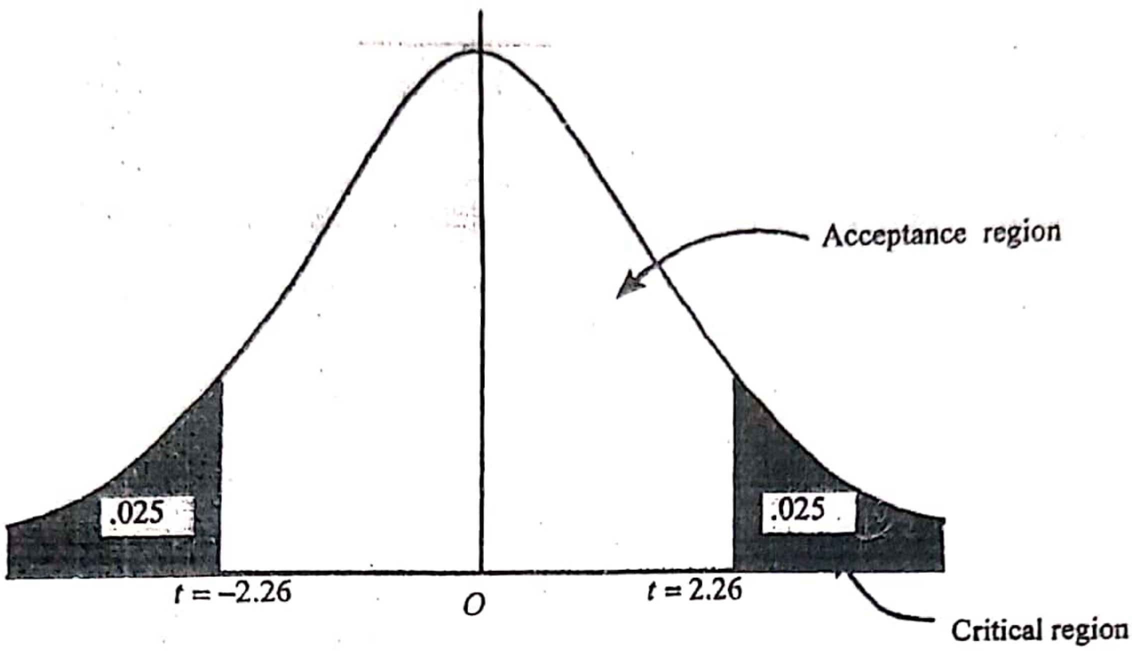
Null Hypothesis: The null hypothesis is $H_0:m_1=m_2$

Alternative hypothesis: The alternative hypothesis is $H_1:m_1\neq m_2$

Test statistic: The suitable test statistic is given by $t = \dfrac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$ whose sampling distribution under H_0 is a t distribution with $n_1 + n_2 - 2$ degrees of freedom, where $s^2 = \dfrac{n_1 S_1^2 + n_2 S_2^2}{n_1 + n_2 - 2}$.

Level of significance: We are to test the null hypothesis at 5% level of significance.

Critical region: Since the alternative hypothesis is both-sided, the critical region is given by $|t| > t_{0.025}$ at 5% significance level. From the given data $t_{0.025} = 2.26$ for 9 degrees of freedom.



Computation: Here $n_1 = 6$, $n_2 = 5$, $\bar{x}_1 = 7.52$, $\bar{x}_2 = 7.49$, $S_1 = 0.024$, $S_2 = 0.032$. The value of s is 0.030869 and therefore the value of t is

$$\frac{7.52 - 7.49}{0.030869\sqrt{\frac{1}{6} + \frac{1}{5}}} = 1.60496.$$

Conclusion: Since $1.60496 \notin \{t : |t| > 2.26\}$ and therefore H_0 is accepted at 5% significance level and conclude that it is not reasonable to believe that the two types of solutions have different pH values.

Q11. Of 400 mangoes selected at random from a large population, 53 were found to be bad. Test at 1% significance level that on the average 10% of the mangoes were bad. Given $\frac{1}{\sqrt{2\pi}}\int_{2.58}^{\infty} e^{-\frac{x^2}{2}}dx = 0.005$. YH – 09, 08

☺. Let p be the probability that a mango selected at random from the given population is bad. If X be the random variable denoting the number of bad mangoes in a random sample of 400 mangoes then X has binomial$(400, p)$ distribution.

Here the observed value of X is $\nu = 53$. Null hypothesis is that the proportion of bad mangoes is 10% i.e. $p = 0.1$

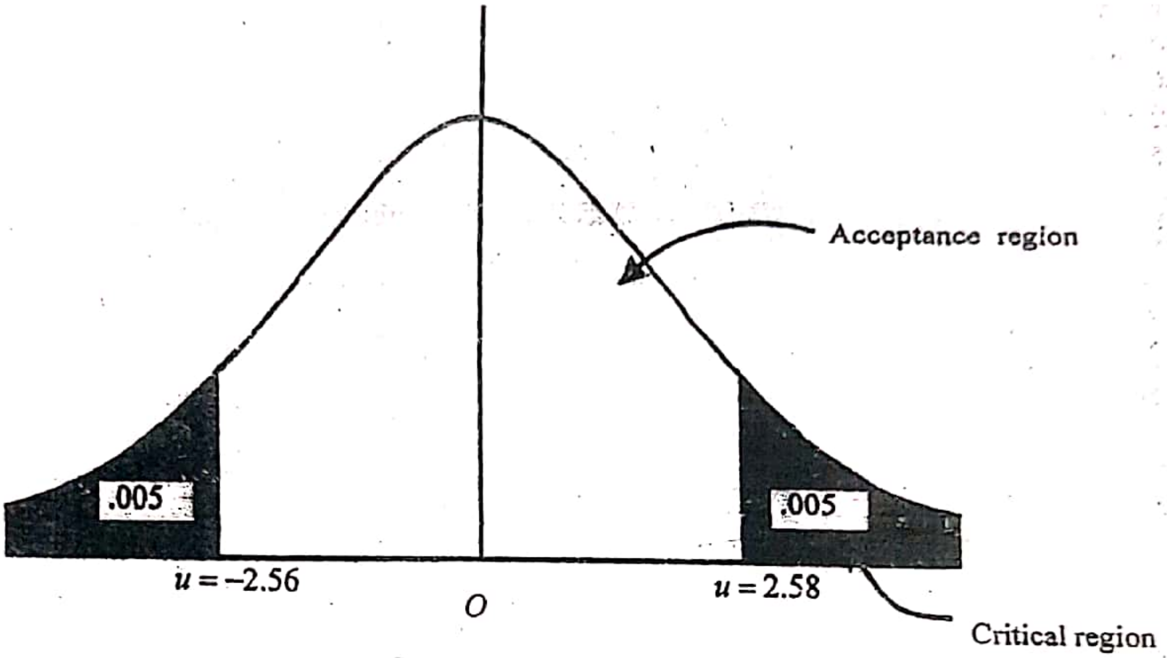
Null Hypothesis: The null hypothesis is that $H_0 : p = 0.1$

Alternative Hypothesis: The alternative hypothesis is that $p \neq 0.1$

Test statistic: The suitable test statistic is given by $\left(u = \frac{\nu - np}{\sqrt{np(1-p)}}\right)$ whose $Z = \frac{P - P_0}{\sqrt{\frac{P_0 q_0}{n}}} \sim N(0,1)$ sampling distribution under H_0 is normal$(0,1)$ if n is large.

Level of significance: We are to test the null hypothesis at 1% significance level.

Critical region: Since the alternative hypothesis is both-sided, the critical region is given by $|u| > u_{0.005}$ at 1% significance level. From the given data $u_{0.005} = 2.58$.



Computation: Here $\nu = 53$, $n = 400$, $p = 0.1$. So the value of u is

$$\frac{53 - 40}{\sqrt{400 \times .1 \times (1 - .1)}} = \frac{13}{6} = 2.17$$

Given $n = 400$, $P = \frac{53}{400} = .1325$

$z = \frac{.1325 - 0.1}{\sqrt{\frac{.1 \times .9}{400}}} = 2.167$

Conclusion: Since $2.17 \notin \{u : |u| > 2.58\}$, therefore H_0 is accepeted at 1% significance level.

Q12. In a sample of 700 males drawn from a large city 400 are smokers and in a sample of 900 males drawn from a large city 400 are smokers.

Do the two cities dpffer significantly in respect of smoking among males ? Test at 1% level of significance. Given $\frac{1}{\sqrt{2\pi}}\int_{2.575}^{\infty} e^{-\frac{x^2}{2}}dx = 0.005$.

Q13. The means of two large samples of 1000 and 2000 members are 67.5 and 68.0 inches respectively. Can the samples be regarded as drawn from the same population ? The population standard deviation is 2.5 inches. Test at 1% level of significance. Given $P(Z > 2.58) = 0.005$.

UP-18

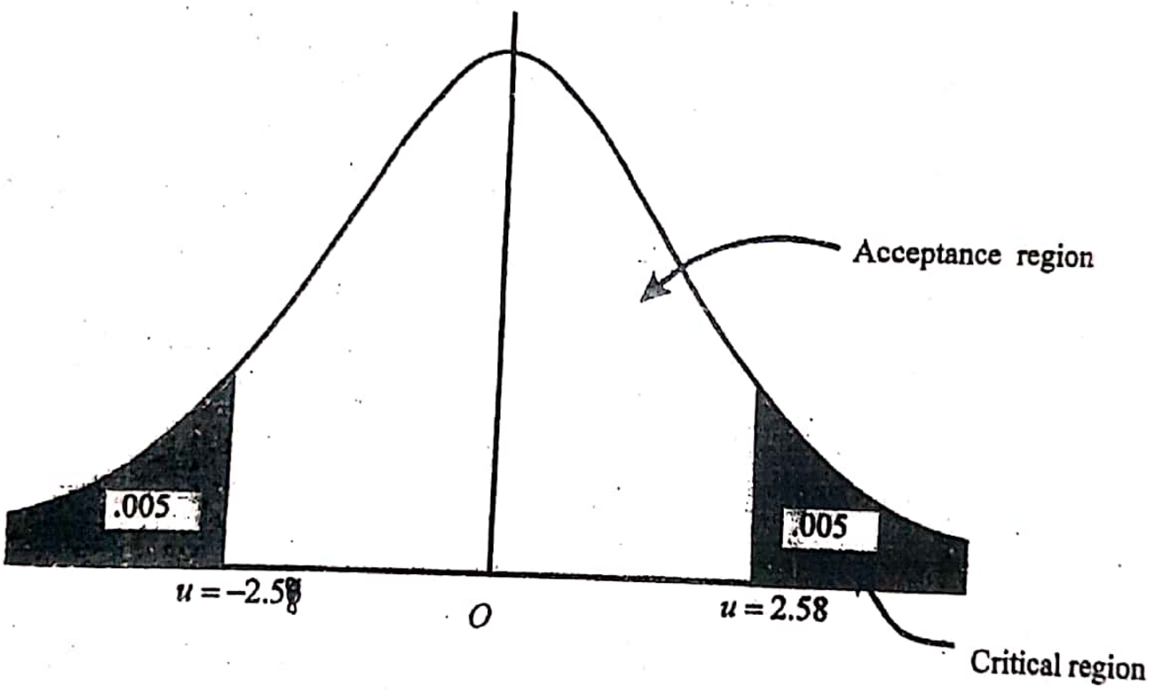
☺. Null Hypothesis: The null hypothesis is that $H_0 : m_1 = m_2$ .i.e., the two means are equal.

Alternative Hypothesis: The alternative hypothesis is that $H_1 : m_1 \neq m_2$ i.e., the two means are not equal.

Test statistic: The suitable test statistic is given by $z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$ whose sampling distribution under H_0 is a normal$(0,1)$.

Level of significance: We are to test the null hypothesis 1% level of significance.

Critical region: Since the alternative hypothesis is two-sided, the critical region is given by $|u| > u_{0.005}$ at 1% significance level. From the given data $u_{0.005} = 2.58$.



Computation: Here $\bar{x}_1 = 67.5$, $\bar{x}_2 = 68.0$, $\sigma = 2.5$, $n_1 = 1000$, $n_2 = 2000$.

So the value of z is $\frac{67.5 - 68.0}{2.5\times\sqrt{\frac{1}{1000}+\frac{1}{2000}}} \simeq -5.2$.

Conclusion: Since $-5.2 \in \{u : |u| > 2.58\}$, therefore we reject the null hypothesis and conclude that the means of the two populations are not equal at 1% significance level.

Q14. A sample of size 10 is drawn from each of the two normal population having the variance which is unknown. If the mean and variance of the sample from the first population are 7 and 26 and those of the sample from the second population are 4 and 10. Test at 5% significance level if the two population have the same mean. [For 18 degrees of freedom $P(t > 2.10) = 0.025$]

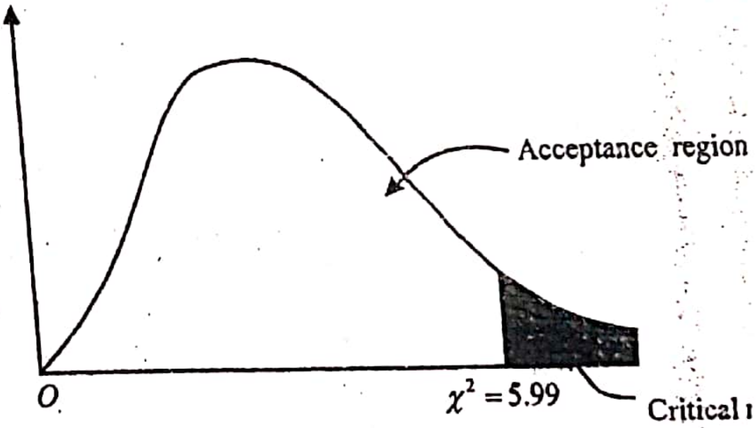
Q15. Of 160 off springs of a certain cross between guinea pigs, 102 were found to be red, 24 black and 34 white. According to a genetic model the probabilities of red, black and white are $\frac{9}{16}, \frac{3}{16}$ and $\frac{1}{4}$ respectively. Test at 5% significance level if the data are consistent with the model, it being given that $P(\chi^2 > 5.99) = 0.05$ for two degrees of freedom.

☺. Null hypothesis is that "the data is consistent with the model"
The expected red, black and white guinea pigs are 90, 30 and 40 respectively.

Observed data (f_0):	102	24	34
Expected data (f_e):	90	30	40
$(f_0 - f_e)^2$:	144	36	36

$$\chi^2 = \sum \frac{(f_0 - f_e)^2}{f_e} = \frac{144}{90} + \frac{36}{30} + \frac{36}{40} \simeq 3.7$$

There are three types, degrees of freedom$=3-1=2$
Since the calculated value of χ^2 i.e., 3.7 is less than the tabulated value 5.99 at 5% level of significance. the conclusion is that the data is consistent with the model.



Q16. A die was thrown 120 times and the frequencies of different faces were observed to be the following:

Face	1	2	3	4	5	6
Observed frequency	25	17	15	23	24	16

Test the hypothesis that the die is fair using a significance level of 0.05. Given $P(\chi^2 > 11.1) = 0.05$ for 5 degrees for freedom.

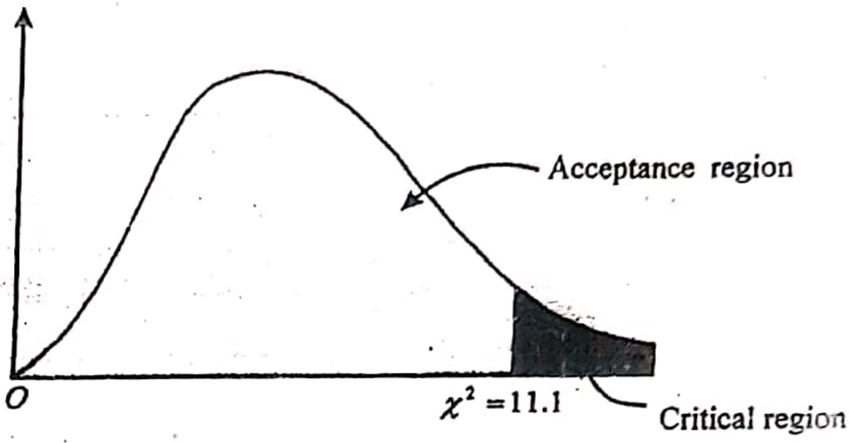
☺. Null hypothesis is that the die is fair. Then the probability of each face is $\frac{1}{6}$ and the expected frequency is $120 \times \frac{1}{6} = 20$ for each.

Face	:	1	2	3	4	5	6
Observed data (f_0):		25	17	15	23	24	16
Expected data (f_e):		20	20	20	20	20	20
$(f_0 - f_e)^2$	:	25	9	25	9	16	16

$$\chi^2 = \sum \frac{(f_0 - f_e)^2}{f_e} = \frac{25}{20} + \frac{9}{20} + \frac{25}{20} + \frac{9}{20} + \frac{16}{20} + \frac{16}{20} \simeq 5$$

There are 6 classes, degrees of freedom$=6-1=5$.

Since the observed value of χ^2 i.e., 5 is less than the tabulated value 11.1 at 5% level of significance, we cannot reject the null hypothesis at 5% level of significance. The conclusion is that the die is fair at 5% level of significance.



Q17. The values of the variable x and the corresponding frequencies f in a sample of size 200 are given below:

Q-20

x	0	1	2	3	4	5	6	7	8	9
f	18	19	23	21	16	25	22	20	21	15

Test the hypothesis that all the values of x in the above are equally likely. [$P(\chi^2 > 14.68) = 0.10$ for 9 degrees of freedom.]

☺. Null hypothesis is that "the values of x are equal probable". Then the probability of each item is $\frac{1}{10}$ and the expected frequency is $200 \times \frac{1}{10} = 20$ for each.

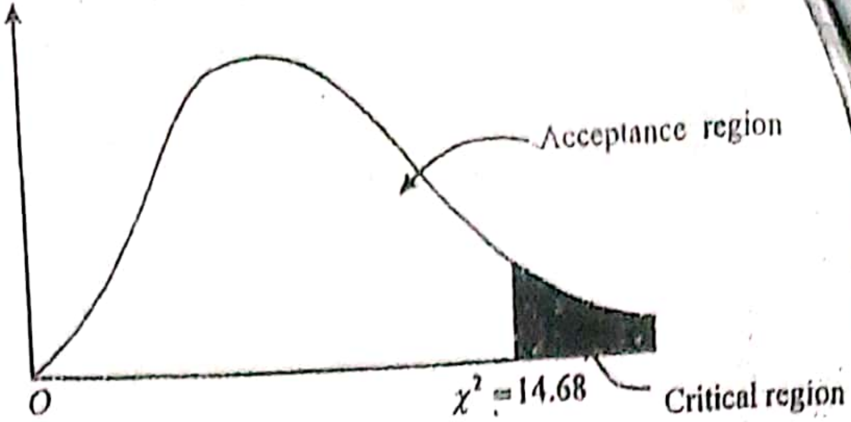
Observed data (f_0):	18	19	23	21	16	25	22	20	21	15
Expected data (f_e):	20	20	20	20	20	20	20	20	20	20
$(f_0 - f_e)^2$:	4	1	9	1	16	25	4	0	1	25

$$\chi^2 = \sum \frac{(f_0 - f_e)^2}{f_e} = \frac{4}{20} + \frac{1}{20} + \frac{9}{20} + \frac{1}{20} + \frac{16}{20} + \frac{25}{20} + \frac{4}{20} + \frac{0}{20} + \frac{1}{20} + \frac{25}{20} = 4.3$$

There are 10 types of values, degrees of freedom$=10-9$.

$= 10 - 1 = 9$

Since the observed value of χ^2 i.e., 4.3 is less than the tabulated value 14.68 at 10% level for 9 degrees of freedom, we cannot reject the null hypothesis at 10% level of significance. The conclusion is that the values of equal probable.



Q18. **The following table gives the number of aircraft accident that occurred during the various days of the week. Test whether the accidents are uniformly distributed over the week using 5% level of significance.**

CP-19

Day	Sun	Mon	Tues	Wed	Thurs	Fri	Sat
No. of accidents	14	14	18	12	11	15	14

Given $P(\chi^2 > 12.59) = 0.05$ for 6 degrees of freedom.

☺. Here we set up the null hypothesis that "the accident are uniformly distributed over the week"

Total number of accident in the week is 98. Therefore the mean accident is $\frac{98}{7} = 14$. Thus the expected frequency of the accidents on each day is 14 under the null hypothesis.

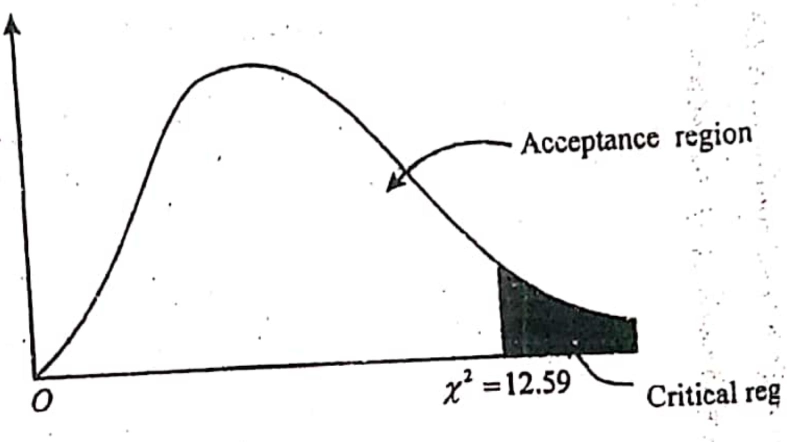
Day	: Sun	Mon	Tues	Wed	Thurs	Fri	Sat
Observed frequency(f_o):	14	14	18	12	11	15	14
Expected frequency(f_e):	14	14	14	14	14	14	14
$(f_o - f_e)^2$:	0	0	16	4	9	1	0

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e} = \frac{0}{14} + \frac{0}{14} + \frac{16}{14} + \frac{4}{14} + \frac{9}{14} + \frac{1}{14} + \frac{0}{14} = \frac{30}{14} = 2.143$$

Degrees of freedom=Number of observation $-1 = 7 - 1 = 6$

$\chi^2_{0.05} = 12.59$ for 6 degrees of freedom.(given)

Since the calculated value of χ^2 is much less than the tabulated value, it is highly in significance and we cannot reject null hypothesis. Hence we conclude that the accidents are uniformly distributed over the week.



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Source: (i) N. G. Das – Statistical Methods
(ii) De & Sen – Statistics