

## 6. Determination of Height and Distance: Theodolite

### What is Theodolite?

A **Theodolite** is a measuring instrument used to measure the horizontal and vertical angles are determined with great precision. Theodolite is more precise than magnetic compass. Magnetic compass measures the angle up to an accuracy of 30'. Anyhow a vernier theodolite measures the angles up to an accuracy of 10'', 20''. It is of either transit or non-transit type. In **Transit theodolites** the telescope can rotate in a complete circle in the vertical plane while Non-transit theodolites are those in which the telescope can rotate only in a semicircle in the vertical plane.



### Types of Theodolite

A

- Transit Theodolite
- Non transit Theodolite

B

- Vernier Theodolite
- Micrometer Theodolite

A

- I. **Transit Theodolite**: a theodolite is called transit theodolite when its telescope can be transited i.e. revolved through a complete revolution about its horizontal axis in the vertical plane.
- II. **Non transit Theodolite**: the telescope cannot be transited. They are inferior in utility and have now become obsolete.

B

- I. **Vernier Theodolite:** For reading the graduated circle if verniers are used, the theodolite is called a vernier theodolite.
- II. Whereas, if a micrometer is provided to read the graduated circle the same is called as a Micrometer Theodolite.

Vernier type theodolites are commonly used.

### **Uses of Theodolite**

Theodolite uses for many purposes, but mainly it is used for measuring angles, scaling points of constructional works. For example, to determine highway points, huge buildings' escalating edges theodolites are used. Depending on the job nature and the accuracy required, theodolite produces more curved of readings, using paradoxical faces and swings or different positions for perfect measuring survey.

### **Followings are the major uses of theodolite:**

- Measurement of the Horizontal and vertical angle
- Measurement of the-magnetic bearing of lines
- Locating points on the line
- Prolonging the survey lines
- Finding difference of level
- Setting out grades
- Ranging curves
- Tachometric survey

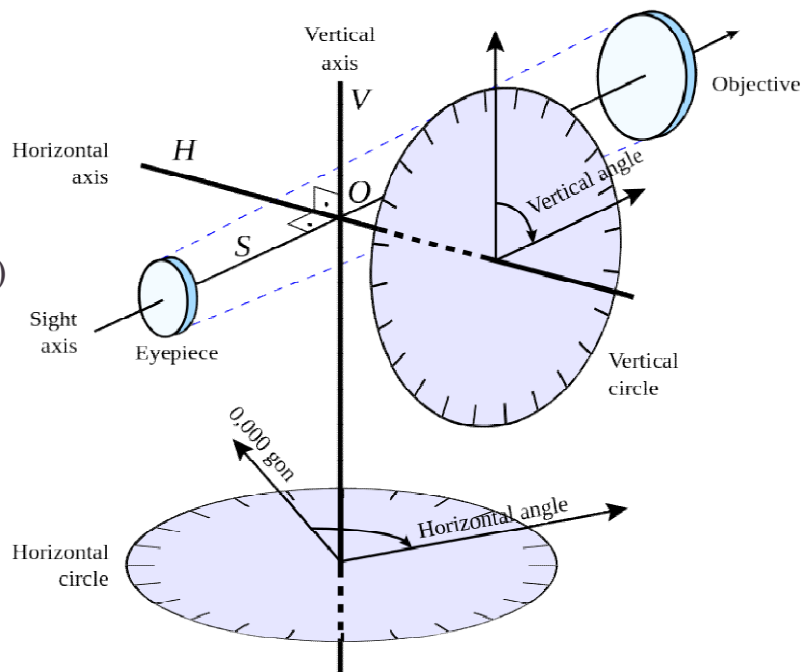
### **Axis of Theodolite**

V – Vertical axis

S – Sight axis, collimation axis

H – Horizontal axis (telescope rotary axis)

L – Level axis (the alidade axis)



## Adjustment of the Instrument

- Temporary Adjustment
- Permanent Adjustment

### Temporary adjustment

Temporary adjustments are a set of operations necessary in order to make a theodolite ready for taking observations at a station. These include its setting up, centering, leveling up and elimination of parallax, and are achieved in four steps:

- **Setting up:** fixing the theodolite onto a tripod along with approximate levelling and centering over the station mark.
- **Centering:** bringing the vertical axis of theodolite immediately over station mark using a centering plate also known as a tribrach.
- **Levelling:** leveling of the base of the instrument to make the vertical axis vertical usually with an in-built bubble-level.
- **Focusing:** removing parallax error by proper focusing of objective and eye-piece. The eye-piece only requires adjustment once at a station. The objective will be re-focused for each subsequent sightings from this station because of the different distances to the target.

### Permanent adjustments:

- **Adjustment of Horizontal plate levels:** The axis of the plate levels must be perpendicular to the vertical axis.
- **Collimation adjustment:** The line of the collimation should coincide with the axis of the telescope and the axis of the objective slide and should be at right angles to the horizontal axis.
- **Horizontal axis adjustment:** the horizontal axis must be perpendicular to the vertical axis.
- **Adjustment of telescope level or the altitude level plate levels:** The axis of the telescope levels or the altitude level must be parallel to the line of collimation.
- **Vertical circle index adjustment:** The vertical circle vernier must read zero when the line of collimation is horizontal.

## Observations with Theodolite

### *Observations with Theodolite*

1. Set up the theodolite over a station at your convenient height with the help of a plumb bob.
2. Release all clamping screws.
3. Rotate the telescope on the vertical plane until the reading on VC and VD is  $0^{\circ}$ – $180^{\circ}$  and clamp it. Rotate it on the horizontal plane to make it parallel to a line joining any two foot-screws.
4. Rotate these two foot screws with both hands simultaneously either inward or outward until the bubble in the level tube is centred.
5. The telescope is then rotated to make it perpendicular to the line joining the previous two foot screws and passing through the remaining foot screws. Rotate this single foot screw until the bubble in the level tube is centred.
6. The three foot screws make an equilateral

triangle. Step 3 to Step 5 is repeated twice separately for the remaining two sides of the triangle. This series of operations make the theodolite perfectly levelled at the given station.

7. Rotate the telescope and look through the eye piece until the line of collimation passes through the station with the staff held vertically over another station. Clamp all the screws of the upper and lower plates and use the slow motion tangential screws for precise sighting.
8. Look through the eye piece, adjust the focussing screw and take the required readings on both the staff and the vernier circles (VC–VD or VA–VD).

*Note:* For measuring an angle, readings must be taken on VC–VD (vertical) or VA–VB (horizontal) for both the left face and the right face. The value of a single angle is thus an average of the four readings. For a reading:

- i. Find the least count (LC) of the main scale, number of vernier divisions (N) and then the vernier constant (VC).
- ii. Take the main scale reading (MSR) and find the vernier division that coincides with the main scale division (i).
- iii. Hence, the required angle =  $(MSR + VC \times i)$ .

## Angle measurement procedure

### Horizontal angle

These angle are used to determine the **bearings** and directions in control surveys, for locating detail when mapping and for setting out all types of structure.

### Vertical angle

These angles are used to determine the heights of points and to calculate slope corrections.

## Vertical Angle Measurement

### To Find the Vertical Angle with a Theodolite

A vertical angle is the angle which the inclined line of sight on the vertical plane makes with the horizontal. It may be either an angle of elevation or an angle of depression. The procedures of measurement are:

- i. The instrument is set up at the given station of observation with precise centering and perfect levelling.
- ii. The telescope is then rotated horizontally until the line of sight bisects the object. Both the vertical and horizontal plates are then clamped. With the help of the vertical slow motion screws, the telescope is moved on the vertical plane to bisect the desired point with great precision.
- iii. The vertical circle readings on both VC and VD are taken for both *face left* and *face right* positions and noted in the field book. The average of these give the required vertical angle (Table 3.8).

**Table 3.8** Field Book

Determination of Angle of Elevation by Transit Theodolite

Date :

Time :

Place :

Inst. No. :

Instrument at	Object Sighted	Face	Vertical Circle Reading		Mean Angle	Grand Mean Angle ( $\alpha$ )
			VC	VD		
X	Top of a building	Left	10°24'	10°28'	10°26'	10°27'
		Right	10°26'	10°30'	10°28'	

# Distance Measurement by Theodolite

To measure ground distance more accurately theodolite is used. Two common methods are

- Stadia Method
- One Degree Method

book are the common observational errors. The natural sources of errors are high wind, high sun, high temperature, curvature and refraction.

### Precautions

At every stage, surveying should be done very carefully. Special precautions should be taken while levelling the instrument, reading the staff, holding the staff vertically over the stations, taking back sight reading at the last station of the former set up in case of change points, checking the office work in detail, etc.

### TRIGONOMETRIC LEVELLING

Trigonometric levelling may be defined as those levelling operations in which relative elevations of different points on the earth's surface are determined from observed *vertical angles* and *horizontal or geodetic distances*. The vertical angles are usually measured with a theodolite while the horizontal distances may either be directly measured with a tape or computed from observations with either a level or a theodolite. Some of the most commonly applied methods are discussed below.

### To Find the Distance between Two Points

On the ground, a distance between two points can be directly measured with a tape or a chain. However to measure it more accurately, a theodolite or a level and a staff may be used. The two common methods are the *stadia-method* and the *one-degree method*. In both of these, the instrument is first set up at one of the points, with accurate centering and perfect levelling. The telescope is then made perfectly horizontal ( $0^\circ - 0^\circ$  setting in VC and VD) and stadia readings are taken on a staff held vertically at the other point.

In the *stadia-method*, either a Dumpy level or a theodolite is needed to take the stadia readings. Distance is then calculated by using the formula:

$$\text{Distance} = [\text{Upper Stadia Reading (USR)} - \text{Lower Stadia Reading (LSR)}] \times \text{Stadia Constant}$$

As per the certification of the instruments (Dumpy level and Theodolite), the value of stadia constant is 100. Therefore,

$$\text{Distance (AB)} = (\text{USR} - \text{LSR}) \times 100.$$

In the *one degree method*, a theodolite is normally used. In this, middle stadia readings (MSR) on a staff held vertically over a point from a station of observation are taken for two particular situations: i) when the telescope is absolutely horizontal and ii) when it is inclined upward by exactly  $1^\circ$ . This is done by rotating the telescope with the help of the vertical slow motion screws and simultaneously looking through the vernier aid in either VC or VD (Fig. 3.34b).

Let,

the MSR (AB) =  $h_1$ , when the telescope is horizontal and

the MSR (AC) =  $h_2$ , when the telescope is  $1^\circ$  inclined upward.

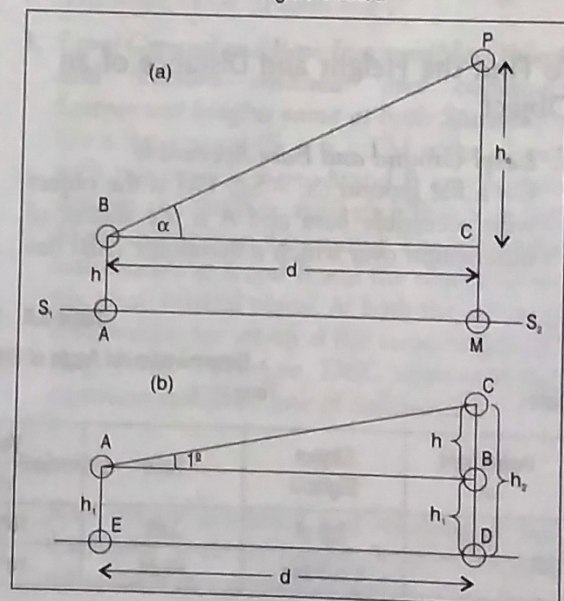
Therefore,

$$BC = h_2 - h_1 \Rightarrow h \text{ (say)}$$

Since  $\angle ABC$  is a rt  $\triangle$ ,

$$\frac{AB}{BC} = \cot 1^\circ$$

Fig. 3.34 Determination of Distance (a) Stadia Method (b) One Degree Method



## Determination of Height and Distance of an Object

1. Level ground and Base Accessible
2. Sloping Ground and Base Accessible
3. Level Ground and Base Inaccessible (Same Instrument height)
4. Level Ground and Base inaccessible (Different Instrument height)

### To Find the Height and Distance of an Object

#### 1. Level Ground and Base Accessible

On a flat ground ( $S_1 - S_2$ ), PM is the object with accessible base and A is the station of observation over which a theodolite (AB) has

been set at a height ( $h$ ) above the ground (Fig. 3.32). Therefore, BC is the line of collimation.

#### Procedure

- i. After proper centering and levelling of the theodolite at A, its height ( $h$ ) is measured with a tape or a graduated staff.
- ii. The horizontal distance, AM ( $d$ ), is then measured either directly with a tape or by computations from observations of the stadia readings on a staff held vertically at the base of the object.
- iii. The angle of elevation ( $\alpha$ ) of the top of the object ( $P$ ) is then found from the observations of the vertical circle readings by the theodolite.

#### Computation

From Fig. 3.34,

$AM \parallel BC$  and  $AB \parallel PM$ .

Therefore,

$CM = AB = h$  and  $BC = AM = d$ .

$d = (USR - LSR) \times \text{Stadia Constant}$

From the rt $\Delta$   $\Delta PBC$ ,

$$PC (h_c) = BC \tan \alpha \\ = d \tan \alpha.$$

Hence,

- i. The horizontal distance of the object from the station of observation is  $d$ ,
- ii. The height of the object above collimation is  $h_c$ ,
- iii. The height of the object above ground is  $(h_c + h)$  and
- iv. The slanting distance (AP) of the object is

$$\sqrt{(h_c + h)^2 + d^2}$$

Fig. 3.34 Determination of Distance (a) Stadia Method (b) One Degree Method

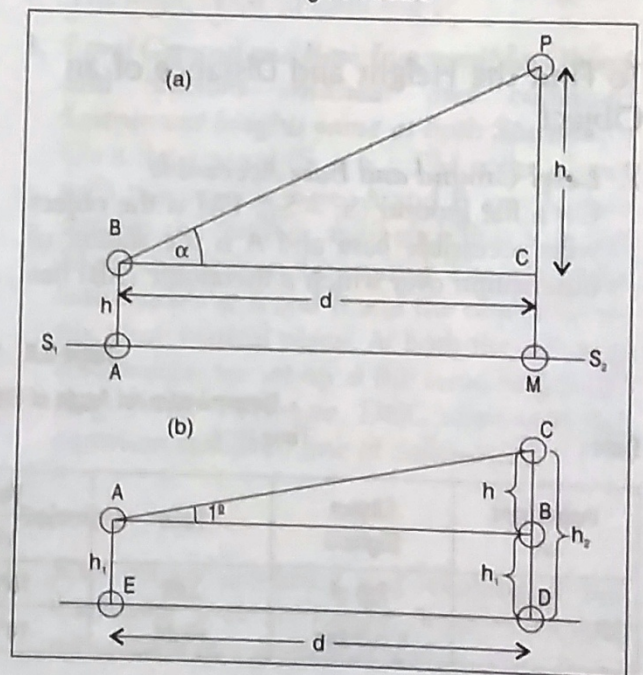
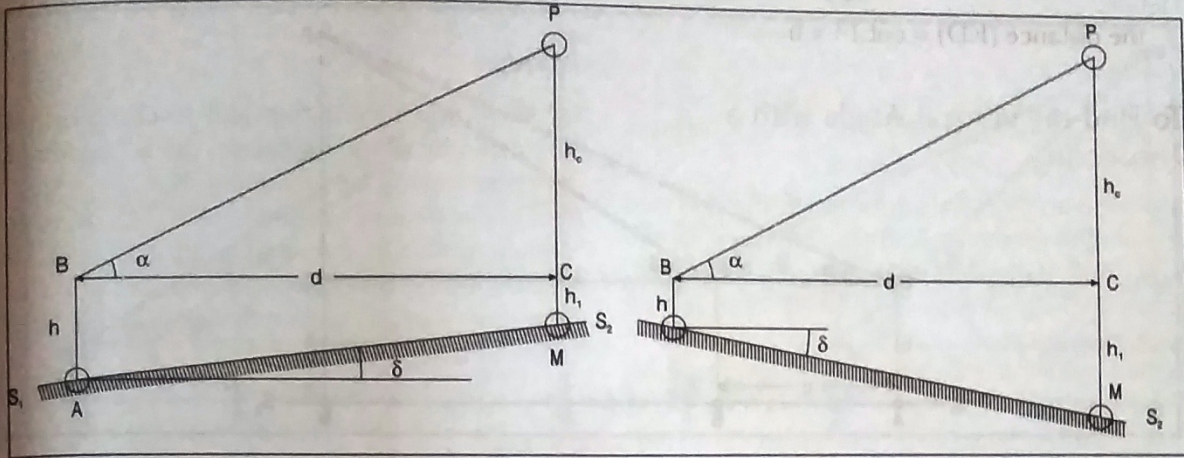


Fig. 3.35 Determination of Height and Distance of an Object



**2. Sloping Ground and Base Accessible**

On a sloping ground ( $S_1 - S_2$ ), PM is the object with an accessible base and A is the station of observation over which a theodolite AB is set up at height, h. Therefore, BC is the line of collimation (Fig. 3.35).

**Procedure**

- i. After proper centering and levelling of the theodolite at A, its height is measured with a tape or a graduated staff.
- ii. The horizontal distance, BC (d), is then measured by observations of the stadia readings on a staff held vertically at the base of the object.
- iii. The angle of elevation ( $\alpha$ ) of the top of the object (P) is then found from the observations of the vertical circle readings by a theodolite.

**Computation**

Horizontal distance,  
 $d = (USR \sim LSR) \times \text{Stadia Constant}$

From Fig. 3.35,  $\Delta PBC$  is rt $\Delta$ . Therefore,

$$PC = BC \cdot \tan \alpha$$

$$= d \cdot \tan \alpha$$

Again,  $\tan \delta = \frac{(h_1 - h)}{d}$

$$\delta = \tan^{-1} \left[ \frac{(h_1 - h)}{d} \right] \text{ where, } \delta = \text{slope}$$

Hence,

- i. The horizontal distance of the object from A = d.
- ii. The height of the object above collimation,  $h_c = PC$ .
- iii. The height of the object above its base =  $(h_c + h_1)$ .
- iv. The height of the object above A =  $(h_c + h)$ .
- v. The slope of the ground =  $\delta^\circ$ .

**3. Level Ground and Base Inaccessible; Object and Stations—collinear and coplaner; Instrument heights same at both Stations.**

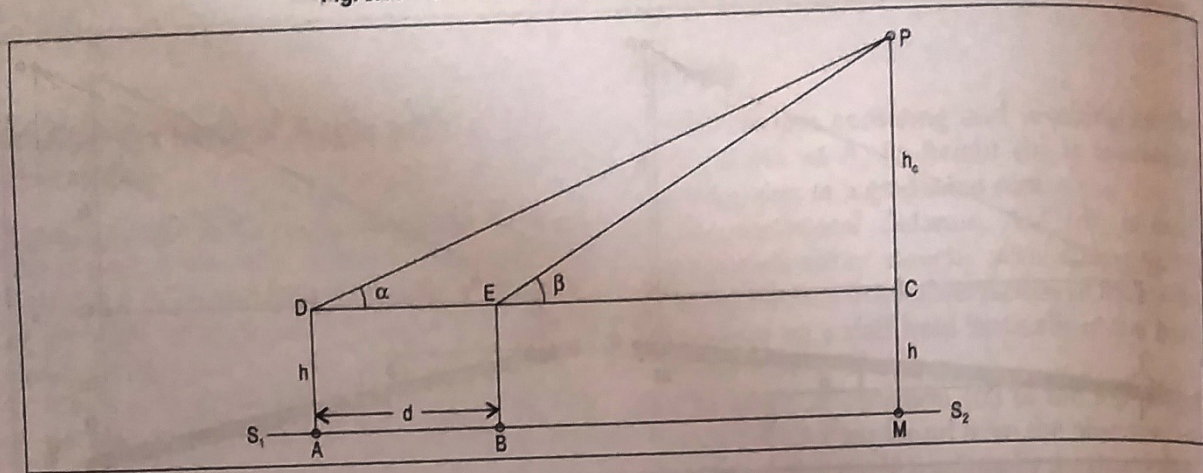
On a flat ground ( $S_1 - S_2$ ), PM is the object with inaccessible base. A and B are the two stations at d distance apart such that A, B and M are collinear on the ground. Therefore, the instruments at A and B and the object lie on the same vertical plane. At both the stations, instruments are set up at the same height (h) (Fig. 3.36). Therefore, DEC represents the common and fixed line of collimation.

**Procedure**

- i. After proper centering and levelling of the theodolite at A, its height (h) is measured with a tape or a graduated staff.



Fig. 3.36 Determination of Height and Distance of an Object



- ii. The angle of elevation ( $\alpha$ ) of the top of the object (P) at A is measured from the observations of the vertical circle readings of a theodolite.
- iii. With the horizontal plate fixed, another station of observation (B) is selected on the ground by looking through the telescope such that  $d$  or  $AB \geq 5$  m.
- iv. The distance,  $AB$  ( $d$ ), is measured with a tape.
- v. The instrument is then shifted to B, set up at height ( $h$ ), carefully centred and levelled and
- vi. The angle of elevation ( $\beta$ ) of the top of the object (P) at B is then found from the vertical circle readings by a theodolite.

### Computation

From Fig. 3.36,

$AM \parallel DC$  and  $AD \parallel BE \parallel MC$ .

Therefore,  $AD = BE = MC = h$  and  $AB = DE = d$ .

From the rt  $\triangle PDC$ ,  $DC = PC \cdot \cot \alpha$  ... (i)

from the rt  $\triangle PEC$ ,  $EC = PC \cdot \cot \beta$  ... (ii)

As D, E and C are collinear and coplaner

$$DC - EC = DE$$

$$\text{or, } PC \cdot \cot \alpha - PC \cdot \cot \beta = d$$

$$\therefore PC (h_c) = \frac{d}{\cot \alpha - \cot \beta}$$

Once  $PC$  is found,  $EC$  can be determined from the equation (ii)

Hence,

- i. The horizontal distance of the object from  $B = EC$  and from  $A = (d + EC)$ .
- ii. The height of the object above collimation,  $h_c = PC$ .
- iii. The height of the object above ground,  $H = (h_c + h)$ .

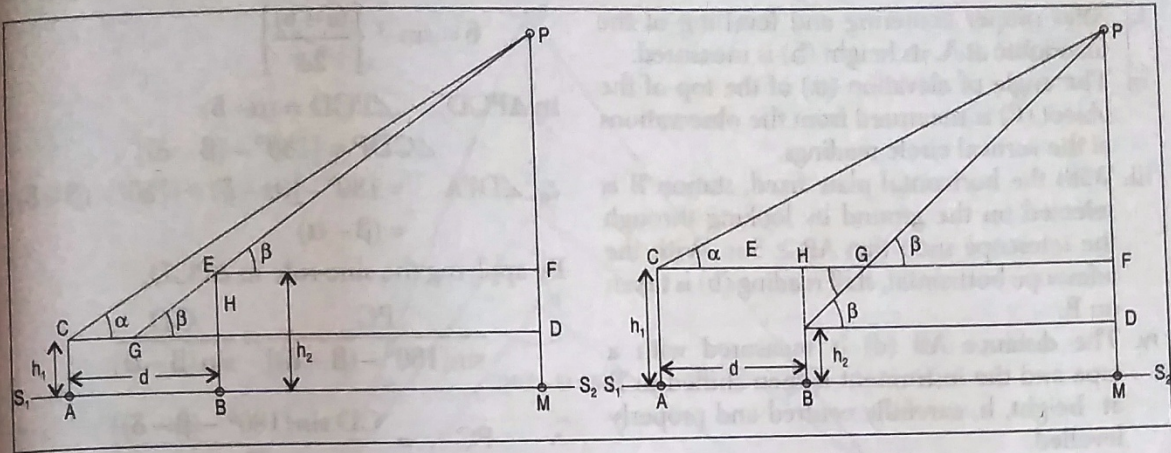
### 4. Level Ground and Base Inaccessible; Object and Stations—collinear and coplaner; Instrument heights different at both Stations.

On a flat ground ( $S_1 - S_2$ ),  $PM$  is the object with inaccessible base. A and B are in the two stations  $d$  distance apart such that A, B and M are collinear on the ground. Instruments are set up at different heights ( $h_1$  at A and  $h_2$  at B) so that the survey operation becomes easier (Fig. 3.37).

### Procedure

- i. After proper centering and levelling of the instrument at A, its height ( $h_1$ ) is measured with a tape or a graduated staff.
- ii. The angle of elevation ( $\alpha$ ) of the top of the object (P) at A is measured from the observations of the vertical circle readings.
- iii. With the horizontal plate fixed, station B is selected on the ground by looking through the telescope such that  $AB$  ( $d$ )  $\geq 5$  m.

Fig. 3.37 Determination of Height and Distance of an Object



- iv. The distance,  $d$  is measured with a tape; the instrument is then set up at  $B$ , at height,  $h_2$ , carefully centred and properly levelled.
- v. The angle of elevation ( $\beta$ ) of the top of the object ( $P$ ) at  $B$  is then measured from the vertical circle readings by a theodolite.

- ii. The height of the object above ground,  $H = (PD + h_1)$ .

5. **Sloping Ground and Base Inaccessible; Object and Stations—Collinear and Coplaner; Instrument Heights Same at Both Stations.**

On a sloping ground ( $S_1 - S_2$ ),  $PM$  is the object with inaccessible base.  $A$  and  $B$  are the two stations at  $d$  distance apart such that  $A$ ,  $B$  and  $M$  are collinear on the ground. Instruments are set up at the same height,  $h$ , on both the stations (Fig. 3.38).

**Computation**

From Fig. 3.37,

$AC \parallel BE \parallel PM$  and  $AM \parallel CD \parallel EF$ .

Therefore,  $AC = BH = MD = h_1$ ,

$BE = MF = h_2$ ,  $AB = CH = d$ ,  $\angle EGH = \angle PEF = \alpha$  and  $EH = EB - HB = h_2 - h_1$ .

From the rt  $\triangle PCD$ ,  $CD = PD \cdot \cot \alpha \dots$  (i)

from the rt  $\triangle PGD$ ,  $GD = PD \cdot \cot \beta \dots$  (ii)

and from the rt.  $\triangle EGH$ ,  
 $GH = EH \cdot \cot \beta \Rightarrow (h_2 - h_1) \cdot \cot \beta \dots$  (iii)

As  $G$  lies on the collimation line  $CD$ ,

$$CD - GD = CG$$

$$\text{or, } PD \cdot \cot \alpha - PD \cot \beta = CH \pm GH$$

[- if  $h_2 > h_1$  and + if  $h_1 > h_2$ ]

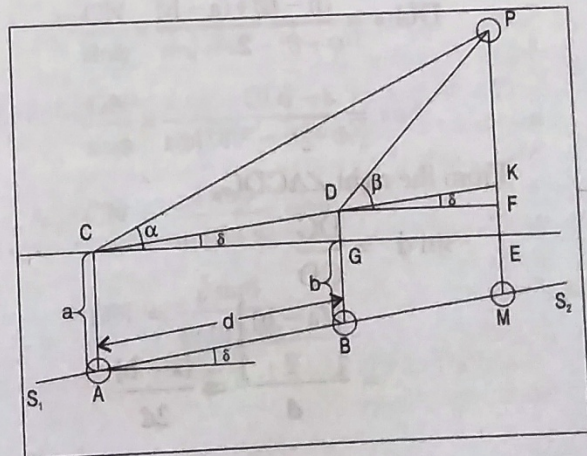
$$\text{or, } PD = \frac{CH \mp GH}{\cot \alpha - \cot \beta}$$

$$= \frac{d \mp (h_2 - h_1) \cot \beta}{\cot \alpha - \cot \beta}$$

Hence,

- i. The horizontal distance of the object from  $A = CD$  and from  $B = (CD - d)$

Fig. 3.38 Determination of Height and Distance of an Object



## Sources of errors in theodolite

The common sources of errors in a theodolite traversing are instrumental, observational and natural.

### Instrumental errors:

- Due to imperfect adjustment of the plate levels.
- **Collimation error**  
The collimation line not being perpendicular to the horizontal axis,
- **Horizontal axis error**  
The horizontal axis not being perpendicular to the vertical axis.
- Non parallelism of the axis of the telescope and the collimation line
- Eccentricity of centres (inner and outer axis)

### Observation Error:

- Inaccurate centering.
- Imperfect levelling
- Wrong manipulation of the tangent screw.
- Wrong setting of the vernier parallax.
- Mistake in reading and recording the values in proper columns of the field book.  
Etc.

### Field or on site errors: ground and weather conditions

- Setting the instrument up on soft ground must be avoided.
- The instrument must be shade when working in hot sunshine.
- If **refraction** is a problem reading must not be taken
- Instrument must be left to adjust to atmospheric conditions
- Observations and setting out angles must not be taken in windy conditions.

### Reference

Sarkar,A., (2015). Practical Geography (3<sup>rd</sup> Ed.) Hyderabad, Telengana, Oriented Black Pvt.Ltd, pp.94-113