Solving with Fuzzy Reasoning for a Green Production–Transportation Supply Chain Model

[a]Kousik Bhattacharya, [b*]Sujit Kumar De

[a,b]*Department of Mathematics, Midnapore College (Autonomous), PIN-721101, India*

a: E-mail:kousikbhattacharya12@gmail.com, b: E-mail: skdemamo2008.com@gmail.com

ARTICLE INFO

Received: 10.07.2021;
Revised: 15.08.2021;
Accepted: 18.08.2021

Key words:

Crude steel production; Pollution; Transportation; Fuzzy reasoning; Modeling; Optimization.

ABSTRACT

In this article we solve a global crude steel production transportation model under the effect of air pollution using fuzzy reasoning technique. First of all, we formulate a cost minimization crisp supply chain model under some specific assumptions which is basically the extension of the article of Bhattacharya et *al.* (2020). Then considering fuzzy system we have shown that, fuzzy reasoning can ultimately optimize the model with respect to some other existing model. Numerical study, graphical illustrations are done to validate the model.

Introduction

Production of crude steel is a noteworthy issue for economic growth of any country in the world. Because of its versatile uses in different sectors, it is now becoming a product of 'all time' uses in our day-to-day activities. But its production process includes several phases requiring major components such as iron ore as raw material, coals for burning and heat generation, water for cooling the system and the uses of high technology for controlling pollution. Karmakar et *al* (2017, 2018) studied extensively to explore the production and production remanufacturing process of a Sponge Iron industry in which they were able to construct functional dependencies of production reproduction quantities with respect to the amount of pollution through the cumulative effect of air, soil and water contamination by the harmful particles containing Sulphur Dioxide (SO_2), Sulphur Trioxide (SO_3), Nitrogen Dioxide (NO_2), Led (Pb), Carbon Di oxide (CO_2), Carbon Monoxide (CO) etc. However, the problem of transporting goods is another critical issue of a production process because it includes fuel costs as well as carbon emission cost. Researchers are basically involved in controlling carbon emission in supply chain (SC) networks (Hernandez-Pellon and Fernandez-Olmo (2018), Ebadi et *al*. (2016), Mancini et *al*. (2016) etc.) in recent times. But the concept of need based production and its corresponding pollution, pollution due to transportation is not studied yet.

At the early stage, most of the models are solved by deterministic and stochastic environment. But they do not obey the realistic models of real-world problems because of the versatile nature of the parameters associated with the models. To overcome the gaps of the deterministic models, researchers were involved to develop the stochastic models where the parameters involved in a model are assumed to be a random number each or every at a time. At the early stage, the operation research (OR) practitioner's/ decision maker's (DM's) problem was how to capture the uncertainty of the several components of any kind of management problems. After the invention of Zadeh (1965)'s fuzzy set theory the situations become quite favorable to all the DM. Since then, numerous research articles have been made to explain and control the complexities of the competitive real-world phenomenon. Some notable works over the EOQ models on fuzzy environments may be pointed out over here. Researchers like De et al. (2014), De and Sana (2015) discussed a deteriorating EOQ model with natural idle time for imprecise demand and in a backlogging model by employing hesitant fuzzy set. The concepts of dense fuzzy set studied by De and Beg (2016) to discuss the frequent learning effect of the fuzzy parameters and contemporarily, the concepts of cloudy fuzzy set were coined by De and Mahata (2017) to enhance the same for continuous time dependent learning effect. Analyzing the behavior of human thinking process De (2017) developed a new fuzzy set namely Triangular dense fuzzy lock set along with its new defuzzification method. After this invention many articles have been made by eminent researchers [De and Mahata (2019a, 2019b), De and Sana (2013a, 2013b, 2018) etc.] to control the individual or group decision making problems on pollution sensitive inventory modelling. The study of fuzzy linguistic dense fuzzy lock set has been discussed by De and Mahata (2019c) recent times to capture the cost effectiveness in trade credit policy. Moreover, researchers like Zadeh (1975), Pal and Mandal (1991) etc. worked over linguistic fuzzy system incorporating the fuzzy approximates reasoning. Fuzzy solutions in fuzzy linear programming problems and preference based on relational fuzzy system were analyzed by Zimmermann (1985), Ramik and Rimanek (1985), Tanaka and Asai (1984) etc.

However, for any kind of optimization problem with some constraints the concept of duality plays an important role to characterize the bounds of the objective values. Ovchinnikov (1991) discussed the duality principle in fuzzy set theory. Wu (2003, 2007) worked extensively over duality theory in fuzzy linear programming problems with fuzzy coefficients, necessity theory and saddle point optimality conditions in fuzzy optimization problems. Zhang et *al*. (2005) studied duality theory in fuzzy mathematical programming problems with fuzzy coefficients. Convex fuzzy mapping with differentiability and its application in fuzzy optimization was discussed by Panigrahi et *al.* (2008). Song and Zhao (2010) developed solution procedures of fuzzy multi-objective optimization of passive suspension parameters. Beyond these concepts, a new approach to duality in fuzzy LPP has been analysed by Nasseri and Ebrahimnejad (2012). To deal with nonlinear fuzzy optimization problems an extension on duality theory was minutely discussed by Zou (2015). Very recent, De et al. (2021a) studied a pollution sensitive model using fuzzy approximate reasoning . Subsequently De et al. (2021b) developed another model and solved it under volumetric fuzzy system.

From the above study it is seen that not a single article has been studied on fuzzy approximate reasoning to design the inventory control problems. Thus, in this article we solve a production transportation model with pollution index under the effect of fuzzy reasoning on the cost parameters of the model. The equivalent crisp problem is constructed by the help of fuzzy alpha cuts and dual fuzzy k cuts for the specific feasible region posed by an approximation index. This article organizes as follows: section 1 includes introduction; Section 2 includes materials and methods describing the concept approximate fuzzy reasoning, dual space, data collection, curve fitting and problem definition. Section 3 discusses the model formulation and solution. Section 4 develops results; section 5 indicates discussion by graphical illustration. Section 6 describes conclusion.

2. Materials and Methods [De (2020)]

This section is designed by some basic definitions of fuzzy membership functions and concepts of fuzzy reasoning. We note that we are going to discuss the minimization problem of the application of OR problems, so we take left fuzzy numbers (L-fuzzy numbers) throughout the article.

2.1 Defining fuzzy membership function

Definition 1: L- fuzzy number: Let us consider the L fuzzy number of the form where and x_0 as initial value of the fuzzy number $\tilde{A}$ and δ is the maximum tolerance

$$\tilde{A} = < x, \mu(x) > \text{ where } \mu(x) = \begin{cases} 1 & if \quad x \geq x_0 \\ 1 - \frac{x_0 - x}{\delta} if & x_0 - \delta \leq x \leq x_0 \\ 0 & if x \leq x_0 - \delta \end{cases}$$

level along with its graphical representations (shown in Fig.-1)

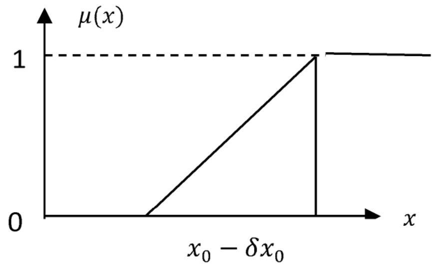
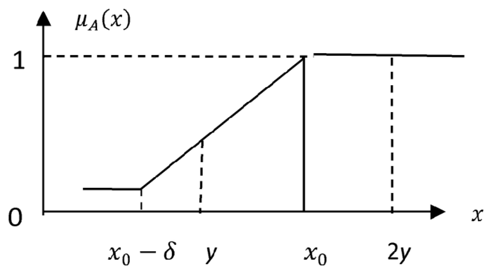


Fig.-1: Membership function of

Definition 2: Let $\tilde{A}$ be the fuzzy number defined as above then its approximated form is denoted as $\mu_A(x)$ (shown in Fig.2) to be obtained from the following possibility formula (obtained from Fig.-3)

$$\mu_A(x) = Poss(\mu_1(x) \cup \mu_2(x)) = \mu_1(x) + \mu_2(x) - \mu_1(x)\mu_2(x)$$

where $\mu_1(x) = \begin{cases} 1 & if x \geq 2y \\ \frac{x-y}{y} & if y \leq x \leq 2y \\ 0 & if x \leq y \end{cases}$ and $\mu_2(x) = \mu_1(x = x_0 - \delta) = \begin{cases} 0 & if x \geq 2y \\ \frac{x_0-\delta}{y} - 1 & if y \leq x \leq 2y \\ 1 & if x \leq y \end{cases}$



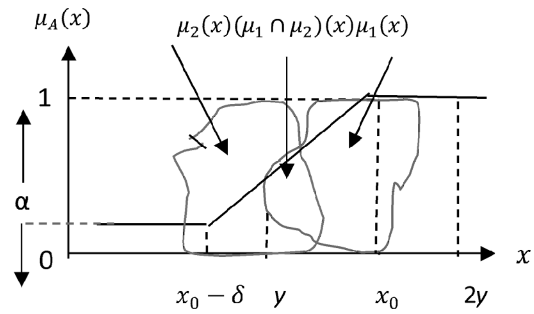


Fig.-2: Membership function of fuzzy reasoning $\tilde{A}$ **Fig.-3:** Approximating membership of $\tilde{A}$

Now from above we get

$$\mu_A(x) = \begin{cases} 1 & if\ x \geq 2y \\ lx - m & if\ y \leq x \leq 2y \\ \frac{x_0-\delta}{y} - 1 & if\ x \leq\ y \end{cases} \quad \text{where} \quad \begin{cases} l = \left(\frac{2}{y} - \frac{x_0-\delta}{y^2}\right) \\ m = 3 - \frac{2(x_0-\delta)}{y} \end{cases}$$

2.2 Defining the Probability density function of the random variable *Y*

Let the probability density function of the random variable *Y* is given by

$p(y) = \frac{2}{\delta}\begin{cases} 1 - \frac{x_0-y}{\delta} & if\ x_0 - \delta \leq y \leq x_0 \\ 0 & otherwise \end{cases}$ such that we always have $\int_{-\propto}^{+\propto} p(y)\, dy = 1$.

2.3 Approximating $\alpha - cut\ of\ \mu_A(x)$

From the fuzzy set $\mu_A(x)$ we construct the $\alpha - cut$ sets as follows: $lx - m \geq \alpha \Rightarrow x \geq \frac{m+\alpha}{l} =$

$\frac{\alpha+3-\frac{2(x_0-\delta)}{y}}{\left(\frac{2}{y}-\frac{x_0-\delta}{y^2}\right)} = \frac{y^2(\alpha+3)-2(x_0-\delta)y}{2y-2(x_0-\delta)}$. The expected value of the $\alpha - cuts\ of\ \mu_A(x)$ is obtained as $E(x) =$ $\frac{(\alpha+3)E(y^2)-2(x_0-\delta)E(y)}{2E(y)-2(x_0-\delta)}$ where, $E(y) = \int_{-\propto}^{+\propto} yp(y)\, dy = \frac{2}{\delta^2}\int_{x_0-\delta}^{x_0} y\{y-(x_0-\delta)\}\, dy = \left(x_0 - \frac{\delta}{3}\right)$

And similarly, $E(y^2) = \frac{2}{\delta^2}\int_{x_0-\delta}^{x_0} y^2\{y-(x_0-\delta)\}\, dy = \left(x_0^2 - \frac{2}{3}\delta x_0 + \frac{\delta^2}{6}\right)$. On substitution we have

$E(x) = \frac{\alpha\left(x_0^2-\frac{2}{3}\delta x_0+\frac{\delta^2}{6}\right)+\left(x_0^2+\frac{2}{3}\delta x_0-\frac{\delta^2}{6}\right)}{x_0+\frac{\delta}{3}}$. The rest part of $\mu_A(x)$ gives $\frac{x_0-\delta}{y} - 1 \leq \alpha \Rightarrow x_0 \leq \delta + (\alpha+1)y$

and giving the expectations $x_0 \leq \delta + (\alpha+1)E(y) \Rightarrow x_0 \leq \delta + (\alpha+1)\left(x_0 - \frac{\delta}{3}\right)$

2.4 Dual β cuts and dual space

Definition 3: Let $\mu_A(x) \geq \alpha$ *with* $0 < \alpha < 1$ be the α–cut of the fuzzy set $\mu_A(x)$ defined on the subset of the universal set X. Let us consider another cut k corresponding to α such that $0 < \alpha\beta < 1$. Then β is said to be dual cut of $\mu_A(x)$ over the convex set $\alpha < 1$.

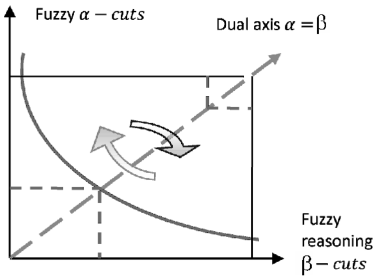


Fig.4: Exchange of dual feasible space

2.5 Approximating feasible space [De(2020)]

Introducing k as approximation parameter (commonly known as approximation index) in such a way that we could be able to get $x_0 \geq \delta$ always and this can be done by approximating x_0 such that $x_0 \geq \delta\left(\frac{2+\alpha\beta}{3\alpha\beta}\right)$ *with* $\alpha\beta < 1$ *and* $\beta < 1$ *where,* α–β together constitute a dual feasible space.

2.6 Problem Definition under Production –Consumption- Pollution scenario

From the above discussion and motivation over the Global Crude Steel production plant model it is seen that Pollution depends upon Production and its transportation. Thus, we may define the research problem in the following way:

i) What will be the actual amount of Global Crude Steel Production such that the aggregate pollution level/ index become minimum?
ii) What should be the actual Consumption level such that the World remains greenery?
iii) How much beneficial the approximate fuzzy reasoning method over traditional optimization method?

2.7 Data collection and curve fitting

Let us consider the secondary data of major 61 crude steel producing countries of the world for the year 2017. We have gathered the data of crude steel production (in terms 10^5 MT), consumption (10^5 MT) (demand), pollution index for 61 countries each. The curve under study is three dimensional, so using production rate (*K*), demand (consumption) rate (*D*) and pollution index (*P*) as three-dimensional variables and taking the help of MATLAB software we fit plane curve in such a way that the R^2 (a statistic) value gets maximum value and the errors assume minimum value. Thus, we obtain equation-1 with its corresponding graphical representation as follows.

$$K - 1.083D = 33.78 - 0.7318\,P \qquad (1)$$

For setting a model we have considered the following inputs (shown in Table 1) which are generally used in several crude steel production sectors in the world.

Table-1: Costs of the proposed model

Set-up cost per cycle $20000	Holding cost per cycle in production plant per MT $5	Deterioration cost per cycle in production plant per unit item $10	Preservation technology cost per cycle $1000
Pollution cost due to production per MT $43.89	Holding cost per cycle in retailer plant per MT $10	Production cost per MT $327.56	Deterioration fraction 0.002
Social cost of carbon per MT $417	Transportation cost per gallon fuel $3.5	Length covered by freight train 600 Miles	Scale parameter for preservation cost (A) 0.00001

3. Model formulation and solution

In this section we formulated our proposed model by considering some notations and assumptions.

3.1. Notations and Assumptions

We may consider the following notations and assumptions for developing the model.

Notations

K Manufacturing rate per cycle (decision variable)

D Retailer's demand rate per cycle

θ Deterioration fraction

ξ Technology uses cost per cycle (\$) ($M(\xi) = \theta(1-e^{-\alpha\xi})$, $0< \alpha<1$)

T_1 Production time (decision variable)

T_2 Inventory opening time of Retailer

T_3 Time at zero inventory in retailer's plant
S_1 Set up cost ($) for supplier's production plant
S_2 Set up cost ($) for retailer's plant
h_1 Holding cost for supplier's plant
h_2 Holding cost for retailer's plant
C_S Global social cost of carbon ($) incurs due to transportation
C_T Transportation cost per unit gallon fuel ($)
C_{Po} Pollution cost per unit item production ($)
C_{Pr} Production cost of unit item ($)
d_C Deterioration cost of unit item ($)
L Transported distance of the produced items
P Pollution index for each country
Q Produced items after time

Assumptions

1. Excessive use of raw materials might destroy the environment.
2. Deteriorated items cannot be recoverable.
3. The time requires for the travel (up and down) of rail freight train equals the manufacturing time per cycle.
4. No deterioration is viewed in the final product during transportation and retailer's inventory.

3.2. Construction of crisp and fuzzy mathematical model [Extension of Bhattacharya et al. (2021)]

a) Supplier's problem

Let the production process starts at supplier's plant with constant production rate K and it continues up to time T_1. During the production run time T_1, some deterioration is viewed which is checked by using modern technology. The inventory level gradually increases and it reaches its maximum value at the end of time T_1. Then finished product gets ready to transport to the retailer at time span $(T_2 - T_1)$. During transportation, the production again starts at zero stock with manufacturing rate K. Due to application of preservation technology, the reduced deterioration is $\delta = \theta - m\ (\xi)$ (say). Therefore, the governing differential equation of supplier's problem is $\frac{dq_1(t)}{dt} = K - \delta q_1(t),\ q_1(0) = 0$ (2)

Solving (2) we get, $q_1(t) = \frac{K}{\delta}\left(1 - e^{-\delta t}\right), \quad 0 \le t \le T_1$ (3)

The inventory holding cost (HCS) for supplier's problem is given by

$HCS = h_1 \int_0^{T_1} q_1(t)dt = \frac{h_1 K}{\delta}\left[T_1 + \frac{e^{-\delta T_1} - 1}{\delta}\right]$ (4)

Then the production cost (PC) is given by $PC = C_{Pr}\left[\int_0^{T_1} q_1(t)dt\right] = \frac{C_{Pr}K}{\delta}\left[T_1 + \frac{e^{-\delta T_1} - 1}{\delta}\right]$ (5)

Also, the deterioration cost (DC) is given by $DC = d_c(KT_1 - Q) = d_c K\left[T_1 - \frac{1 - e^{-\delta T_1}}{\delta}\right]$ (6)

Where $Q = \frac{K}{\delta}\left(1 - e^{-\delta T_1}\right) = D(T_3 - T_2)$ (7)

The transportation cost (TC) is given by $TC = C_T \times \frac{2LQ}{471} = 0.00424628LQC_T$ (8)

Pollution cost due to transportation (TPC) is given by

$TPC = C_S \times \frac{2LQ}{471} \times 22.38 \times 0.000454 = 0.0000431445LQC_S$ (9)

Pollution cost due to production (PPC) is given by $PPC = C_{Po} \times KT_1$ (10)

The set-up cost (SCS) for supplier's problem is given by $SCS = S_1$ (11)

b) Retailer's problem

At the time T_2, the items are received and began to sale at the retailer's counter and the inventory will get at time T_3 because of demand *D*. During the interval $[T_2, T_3]$, the variation in the inventory depletes for demand only. Therefore, the governing differential equation of retailer's problem is

$\frac{dq_2(t)}{dt} = -D, \quad T_2 \le t \le T_3, \quad q_2(T_3) = 0$ (12)

Which gives $q_2(t) = D(T_3 - t), \quad T_2 \le t \le T_3$ (13)

The inventory holding cost (HCR) for retailer's problem is given by

$HCR = h_2 \int_{T_2}^{T_3} q_2(t)dt = \frac{h_2 D}{2}(T_3 - T_2)^2$ (14)

The set-up cost (SCR) for retailer's problem is given by $SCR = S_2$ (15)

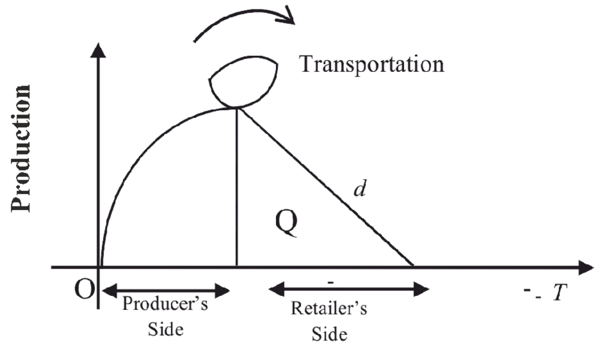


Fig. 5: Production-Transportation Model

Combining supplier's problem and retailer's problem, the total average joint inventory system cost (z) is given by

$$z = \frac{1}{T_1}\left\{\frac{h_1 K}{\delta}\left(T_1 + \frac{e^{-\delta T_1}-1}{\delta}\right) + d_c K\left(T_1 - \frac{1-e^{-\delta T_1}}{\delta}\right) + \frac{C_{Prod}K}{\delta}\left(T_1 + \frac{e^{-\delta T_1}-1}{\delta}\right) + C_{Pol} \times KT_1 + S_1\right\} + \frac{1}{2(T_2-T_1)}\{C_T \times 0.00424628LQ + C_S \times 0.0000431445LQ\} + \frac{1}{(T_3-T_2)}\left\{\frac{h_2 D(T_3-T_2)^2}{2} + S_2\right\} \quad (16)$$

Therefore, the basic mathematical problem of the model is obtained as

$$\begin{cases} Minimize\ z(K, T_1, Q, P, D) \\ subject\ to, T_2 = \frac{3}{2}T_1, T_3 = \frac{5}{2}T_1 \\ Q = \frac{K}{\delta}\left(1 - e^{-\delta T_1}\right) = DT_1 \\ K - 1.083D = 33.78 - 0.7318P \end{cases} \quad (17)$$

This can be defined in another form as

$$\begin{cases} Minimize\ z = F_1 + F_2 \\ subject\ to \\ Q = \frac{K}{\delta}\left(1 - e^{-\delta T_1}\right) = DT_1 \\ K - 1.083D = 33.78 - 0.7318P \end{cases} \quad (18)$$

With the condition:

$$\begin{cases} F_1 = c_1 f_1 + c_2 f_2 + c_3 f_3 + c_4 f_4 + c_5 f_5 + c_6 f_6 + c_7 f_7 \\ F_2 = c_8 f_8 + c_9 f_9 \\ (c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9) \equiv \left(h_1, d_c, c_{pd}, c_{po}, s_1, c_T, c_s, h_2, s_2\right) \\ f_1 = \frac{K}{\delta T_1}\left(T_1 - \frac{1-e^{-\delta T_1}}{\delta}\right), f_2 = \frac{K\left(T_1 - \frac{1-e^{-\delta T_1}}{\delta}\right)}{T_1} \\ f_3 = \frac{K}{\delta T_1}\left(T_1 - \frac{1-e^{-\delta T_1}}{\delta}\right), f_4 = K, f_5 = \frac{1}{T_1} \\ f_6 = \frac{0.00424628LK\left(1-e^{-\delta T_1}\right)}{T_1\delta}, f_7 = \frac{0.0000431445LK\left(1-e^{-\delta T_1}\right)}{T_1\delta} \\ f_8 = \frac{DT_1}{2}, f_9 = \frac{1}{T_1} \end{cases} \quad (19)$$

Now, we consider all the cost parameters associated with the model might follow triangular fuzzy number as defined in the section 2 then the equivalent fuzzy problem of the proposed model is as below

$$\begin{cases} Minimize\ \widetilde{Z} = \widetilde{F_1} + \widetilde{F_2} \\ Subject\ to,\ Q = \frac{K}{\delta}\left(1 - e^{-\delta T_1}\right) = DT_1 \\ K - 1.083D = 33.78 - 0.7318P \end{cases} \tag{20}$$

$$\text{where} \begin{cases} \widetilde{F_1} = \widetilde{c_1}\, f_1 + \widetilde{c_2}\, f_2 + \widetilde{c_3}\, f_3 + \widetilde{c_4}\, f_4 + \widetilde{c_5}\, f_5 + \widetilde{c_6}\, f_6 + \widetilde{c_7}\, f_7 \\ \widetilde{F_2} = \widetilde{c_8}\, f_8 + \widetilde{c_9}\, f_9 \end{cases} \tag{21}$$

For defuzzification of the problem (20) using the concepts of approximate fuzzy reasoning studied at subsections (2.3- 2.5) and α – cuts of fuzzy numbers the equivalent crisp problem is given by

$$\begin{cases} Maximize\ \alpha,\ 0 < \alpha < 1 \\ Subject\ to\quad E(Z_\alpha) \le Z_0 + (1-\alpha)\delta_0 \\ c_{i0} \ge \delta_{1i}\left(\frac{2+\alpha\beta}{3\alpha\beta}\right),\quad i = 1,2,\dots,9 \\ \alpha\beta < 1\ ,\ \beta < 1 \\ Q = \frac{K}{\delta}\left(1 - e^{-\delta T_1}\right) = DT_1, K - 1.083D = 33.78 - 0.7318P \end{cases} \tag{22}$$

$$\begin{cases} E(Z_\alpha) = c_{1\alpha}\, f_1 + c_{2\alpha}\, f_2 + c_{3\alpha}\, f_3 + c_{4\alpha}\, f_4 + c_{5\alpha}\, f_5 + c_{6\alpha}\, f_6 + c_{7\alpha}\, f_7 + c_{8\alpha}\, f_8 + c_{9\alpha}\, f_9 \\ c_{i\alpha} = \dfrac{\alpha\left(c_{i0}^2 - \frac{2}{3}\delta_{1i}c_{i0} + \frac{\delta_{1i}{}^2}{6}\right) + \left(c_{i0}^2 + \frac{2}{3}\delta_{1i}c_{i0} - \frac{\delta_{1i}{}^2}{6}\right)}{c_{i0} + \frac{\delta_{1i}}{3}}, i = 1,2,\dots,9 \\ f_1 = \frac{K}{\delta T_1}\left(T_1 - \frac{1-e^{-\delta T_1}}{\delta}\right), f_2 = \dfrac{K\left(T_1 - \frac{1-e^{-\delta T_1}}{\delta}\right)}{T_1}, f_3 = \frac{K}{\delta T_1}\left(T_1 - \frac{1-e^{-\delta T_1}}{\delta}\right), f_4 = K, f_5 = \frac{1}{T_1} \\ f_6 = \frac{0.00424628LK\left(1-e^{-\delta T_1}\right)}{T_1\delta}, f_7 = \frac{0.0000431445LK\left(1-e^{-\delta T_1}\right)}{T_1\delta}, f_8 = \frac{DT_1}{2}, f_9 = \frac{1}{T_1} \end{cases} \tag{23}$$

4. Results

In this section we illustrate the novelty of the methodology by using numerical data collected from a crude steel industry.

4.1 Crisp optimal solution

Using this data set stated in Table 1 and utilizing the pollution function (1) in the main problem (18) with the condition (19) we get the optimal results by taking the help of LINGO software and they are put in Table 2.

Table-2: Crisp solution of the proposed model

P^*	K^*	D^*	T_1^*	T_2^*	T_3^*	Q^*	Z^*
26	280.57	245.44	1.45	2.18	3.625	355.89	48342.68

Table 2 shows the optimal average inventory cost $ 48342.68 with the optimal order quantity 355.89 MT, production time 1.45 months. The optimal production rate and demand rate is 280.57 MT and 245.44 MT. In this case the pollution index takes the value explicitly.

4.2 Optimal solution using fuzzy approximate reasoning method

For finding fuzzy optimal solution under the effect of fuzzy approximate reasoning we consider the values of the fuzzy deviation parameters δ_0=9000, δ_{10}=1, δ_{20}=2,δ_{30}=60,δ_{40}= 9,δ_{50}=2500,δ_{60} = 0.7,δ_{70}= 80,δ_{80}=2,δ_{90}= 2500, z_0 = 44018 along with P=26,K=280.57, D= 245.44, T_1=1.45. The fuzzy optimal solutions are tabulated in table 3.

Table 3: Optimal solution using approximate fuzzy reasoning method

α^*	β^*	Q^*	T_1^*	Z^*
0.21	0.99	302.201	1.23	46621.37
0.22	0.95	245.44	1.00	46523.54
0.23	0.90	308.215	1.26	46412.74
0.25	0.85	311.790	1.27	46288.90
0.26	0.80	245.44	1.00	46149.58
0.28	0.75	245.44	1.00	45991.68
0.30	0.70	325.608	1.33	45811.23
0.32	0.65	331.642	1.35	45603.02
0.35	0.60	338.684	1.38	45360.11
0.38	0.55	347.01	1.41	45073.02
0.42	0.50	356.99	1.45	44728.53
0.47	0.45	302.953	1.23	44307.47
0.36	0.581	341.649	1.39	45257.85
0.34	0.611	245.44	1.00	45419.27

Table 3 expresses the optimal average inventory cost, optimum order quantity and optimum production run time with respect to different α cuts and its dual cuts β. We see that the average inventory cost takes minimum value \$ 44307.47 with respect to order quantity 302.953 MT and production cycle 1.23 months for the primal-dual cut value (0.47, 0.45) and the average inventory cost takes its maximum value \$ 46621.37 with respect to order quantity 302.201 MT and same production cycle 1.23 months for the primal-dual cut value (0.21, 0.99).

4.3. Sensitivity Analysis

To investigate the sensitivity of the parameters associated with the problem we take the changes of the cost parameters as well as fuzzy deviation parameters from -20% to +20% each. The optimal production run time, order quantity, average inventory cost and the primal-dual cut value are furnished in Table 4A and 4B.

Table 4A: Sensitivity analysis of fuzzy cost parameters (optimum α)

Parameter	% Change	α*	β*	Q*	T1*	Z*	$\frac{z-z^*}{z^*}\times 100\%$
C_{10}	+20	0.309	0.588	245.44	1.00	50233.55	3.91
	+10	0.519	0.350	401.798	1.64	48342.68	0
	-10	0.320	0.569	346.861	1.41	50140.26	3.72
	-20	0.519	0.350	355.896	1.45	48342.68	0
C_{20}	+20	0.309	0.588	245.44	1.00	50233.55	3.91
	+10	0.519	0.350	355.896	1.45	48342.68	0
	-10	0.519	0.350	401.798	1.64	48342.68	0
	-20	0.519	0.350	355.896	1.45	48342.68	0
C_{30}	+20	0.519	0.35	355.896	1.45	48342.68	0
	+10	0.309	0.588	245.44	1.00	50233.55	3.91
	-10	0.519	0.35	401.798	1.64	48342.68	0
	-20	0.519	0.35	401.798	1.64	48342.68	0
C_{40}	+20	0.272	0.668	263.876	1.08	50568.67	4.6
	+10	0.288	0.631	253.262	1.03	50425.48	4.31
	-10	0.519	0.35	389.782	1.59	48342.68	0
	-20	0.396	0.471	245.44	1.00	49449.71	2.29
C_{50}	+20	0.519	0.35	450.651	1.84	48342.68	0
	+10	0.519	0.35	421.457	1.72	48342.68	0
	-10	0.583	0.35	381.736	1.56	47770.19	-1.18
	-20	0.664	0.35	548.024	2.23	47037.93	-2.7

C_{60}	+20	0.299	0.609	246.852	1.01	50331.41	4.11
	+10	0.519	0.35	362.747	1.48	48342.68	0
	-10	0.519	0.35	399.642	1.63	48342.68	0
	-20	0.519	0.35	343.042	1.40	48342.68	0
C_{70}	+20	0.519	0.35	373.027	1.52	48342.68	0
	+10	0.301	0.604	245.501	1.00	50310.74	4.07
	-10	0.519	0.35	347.988	1.42	48342.68	0
	-20	0.519	0.35	340.470	1.39	48342.68	0
C_{80}	+20	0.301	0.603	245.44	1.00	50306.36	4.06
	+10	0.519	0.35	361.532	1.47	48342.68	0
	-10	0.519	0.35	401.799	1.64	48342.68	0
	-20	0.318	0.35	245.44	1.00	50159.75	3.76
C_{90}	+20	0.519	0.35	450.651	1.84	48342.68	0
	+10	0.519	0.35	421.457	1.72	50141.64	3.72
	-10	0.343	0.594	245.44	1.00	48342.68	0
	-20	0.664	0.35	548.023	2.23	50142.47	3.72
δ_0	+20	0.519	0.350	339.393	1.38	49207.61	1.79
	+10	0.325	0.560	245.44	1.00	50703.14	4.88
	-10	0.302	0.602	248.79	1.01	49673.19	2.75
	-20	0.519	0.35	374.37	1.53	47477.74	-1.79

Table 4A shows the sensitivity of the cost parameters associated with the model. We see that the average inventory cost reaches its upper bound \$ 50703.14 with 4.88 % increment and takes its lower bound \$ 47037.93 with 2.7 % decrement. The order quantity and the production run time lie between the respective intervals [245.44, 548.02] MT and [1.00, 2.23] months. Also, the primal α cut lies in the interval [0.272, 0.664] whereas the dual β cut lies in the interval [0.350, 0.668] throughout the table.

Table 4B: Sensitivity analysis of fuzzy deviation parameters (optimum α)

Parameter	% Change	α*	β*	Q*	T^*_1	Z*	$\frac{z-z^*}{z^*}\times 100\%$
C_{10}	+20	0.519	0.35	355.896	1.45	48342.68	0
	+10	0.320	0.569	346.819	1.41	50141.64	3.72
	-10	0.519	0.35	401.798	1.64	48342.68	0
	-20	0.319	0.569	346.794	1.41	50142.47	3.72
C_{20}	+20	0.320	0.569	346.811	1.41	50141.91	3.72
	+10	0.320	0.569	346.811	1.41	50141.91	3.72
	-10	0.320	0.569	346.811	1.41	50141.91	3.72
	-20	0.519	0.35	355.896	1.45	48342.68	0
C_{30}	+20	0.519	0.35	401.798	1.64	48342.68	0
	+10	0.519	0.35	401.798	1.64	48342.68	0
	-10	0.318	0.572	346.295	1.41	50158.73	3.76
	-20	0.519	0.350	355.896	1.45	48342.68	0
C_{40}	+20	0.519	0.35	401.164	1.63	48342.68	0
	+10	0.519	0.35	401.482	1.64	48342.68	0
	-10	0.519	0.35	402.114	1.64	48342.68	0
	-20	0.309	0.588	245.44	1.00	50235.55	3.92
C_{50}	+20	0.635	0.35	457.689	1.86	47303.71	-2.15
	+10	0.577	0.35	400.204	1.63	47827.96	-1.06
	-10	0.519	0.35	402.183	1.64	48342.68	0
	-20	0.519	0.35	402.565	1.64	48342.68	0
C_{60}	+20	0.519	0.35	401.688	1.64	48342.68	0
	+10	0.519	0.35	401.744	1.64	48342.68	0
	-10	0.519	0.35	356.065	1.45	48342.68	0
	-20	0.519	0.35	356.235	1.45	48342.68	0
C_{70}	+20	0.519	0.35	355.501	1.45	48342.68	0
	+10	0.519	0.35	355.698	1.45	48342.68	0
	-10	0.519	0.35	356.093	1.45	48342.68	0
	-20	0.519	0.35	401.925	1.64	48342.68	0
C_{80}	+20	0.519	0.35	355.619	1.45	48342.68	0
	+10	0.519	0.35	355.758	1.45	48342.68	0
	-10	0.519	0.35	356.034	1.45	48342.68	0
	-20	0.569	0.319	346.781	1.41	50142.90	3.72
C_{90}	+20	0.635	0.35	457.689	1.86	47303.71	-2.15
	+10	0.317	0.637	248.363	1.01	50165.41	3.77
	-10	0.519	0.35	402.183	1.64	48342.68	0
	-20	0.519	0.35	402.565	1.64	48342.68	0
δ_0	+20	0.519	0.350	245.44	1.00	57146.28	18.21
	+10	0.420	0.432	245.44	1.00	53634.74	10.95
	-10	0.268	0.679	277.55	1.13	46207.85	-4.42
	-20	0.228	0.797	324.62	1.32	42162.54	-12.78

Table 4B shows the sensitivity of the deviation parameters associated with the model. We see that the average inventory cost reaches its upper bound \$ 57146.28 with 18.21 % increment and takes its lower bound \$ 42162.54 with 12.78 % decrement. The order quantity and the production run time lie between the respective intervals [245.44, 457.69] MT and [1.00, 1.86] months. Also, the primal α cut lies in the interval [0.228, 0.635] whereas the dual β cut lies in the interval [0.319, 0.797] throughout the table.

Table 5: Variation of supply chain cost with rail distance in fuzzy approximate reasoning

L	$\alpha*$	$\beta*$	$Q*$	T_1*	$Z*$
500	0.519	0.35	394.092	1.606	48342.68
600	0.519	0.35	355.896	1.45	48342.68
700	0.319	0.569	353.479	1.44	50138.80

Table 5 shows the variation of average inventory cost, order quantity, production run time and the primal-dual cuts with respect to the variation of distance covered by the transportation vehicles. We see that the average inventory cost is increasing and both the order quantity and production run time are decreasing with respect to the increasing transportation distances.

5. Discussion

In this section we shall draw some graphs using numerical outcomes of the model stated in tables 2 to 5.

Fig. 6: Change of order quantity due to variation of z_0, δ_0 and δ_{20}

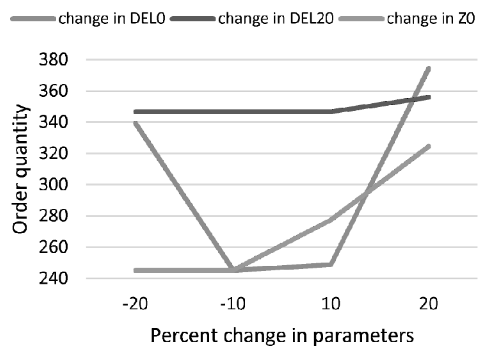


Fig. 7: Change of production run time due to variation of z_0, δ_0 and δ_{20}

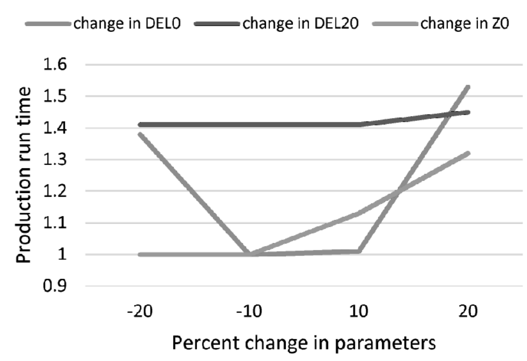


Fig. 6 and Fig. 7 show the variation of the order quantity and production run time due to the % change of the parameters z_0, δ_0 and δ_{20}. We see that δ_{20} is less sensitive and z_0, δ_0 are much sensitive according to the order quantity curve and production run time curve. The order quantity takes maximum value at 375 MT for 20 % hike of δ_0 and takes minimum value at 245 MT for 10% decrease of both z_0 and δ_0. The production run time reaches its maximum at 1.55 months for 20 % hike of δ_0 and takes minimum value at 1.00 months for 10% decrease of both z_0 and δ_0.

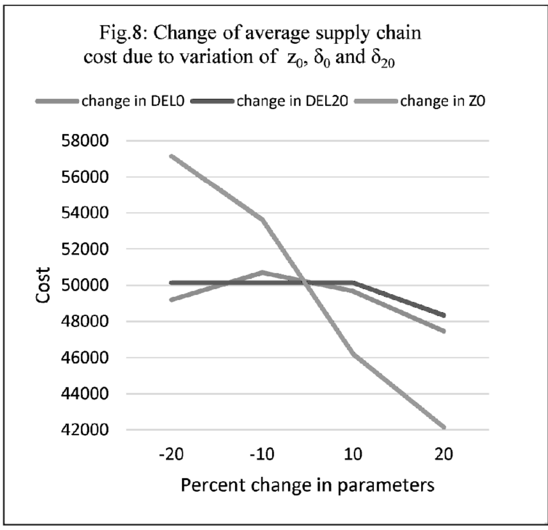


Fig.8: Change of average supply chain cost due to variation of z_0, δ_0 and δ_{20}

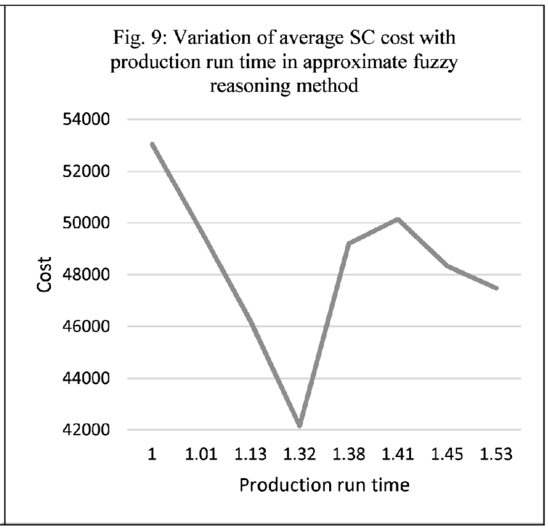


Fig. 9: Variation of average SC cost with production run time in approximate fuzzy reasoning method

Fig. 8 shows the variation of the average supply chain cost due to the % change of the parameters z_0, δ_0 and δ_{20}. We see that z_0 is much sensitive rather than δ_0 and δ_{20} according to the cost curve. The average supply chain cost becomes maximum ($ 57000) with 20% reduction of z_0 and becomes minimum ($ 42000) with 20 % hike of z_0. Fig. 9 gives the variation of average SC cost with the variation of production run time. The cost curve takes V-shape with valley $42000 corresponding to 1.32 months production time and takes two peaks with $ 53000 and $ 50000 corresponding to 1.00 months and 1.41 months respectively.

Fig. 10:Variation of Supply Chain cost under α-dual β–cuts solutions

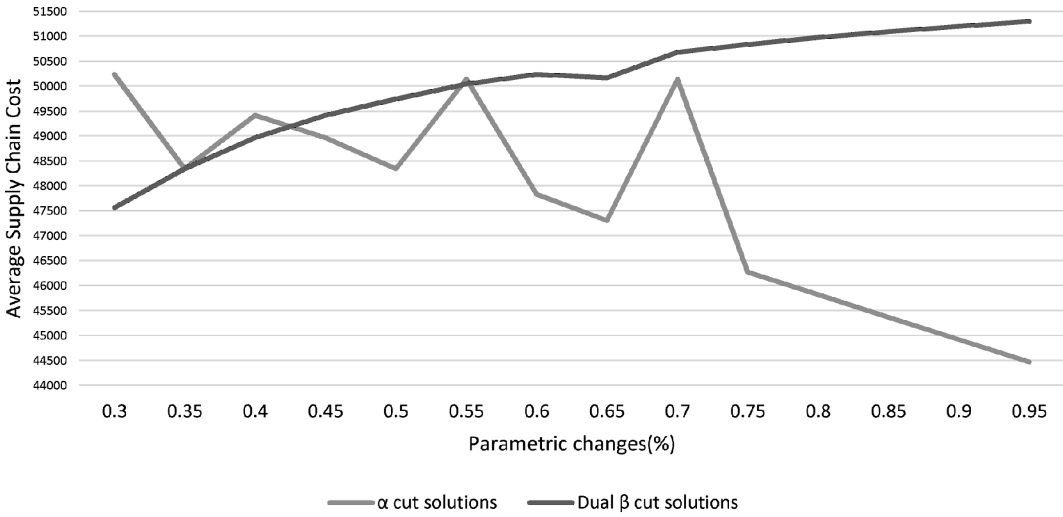


Fig. 10 represents the variation of the average supply chain cost under primal α-dual β–cuts solutions. We see that the average SC cost curve takes a parabolic view with respect to the dual cut whereas it takes a zigzag view with respect to the primal cut. When the primal cut becomes maximum with 0.95 then the average SC cost becomes minimum with $ 44500 and when the dual cut becomes maximum with 0.95 then the average SC cost becomes maximum with $ 51000.

Fig. 11: Variation of cost function due to α and β

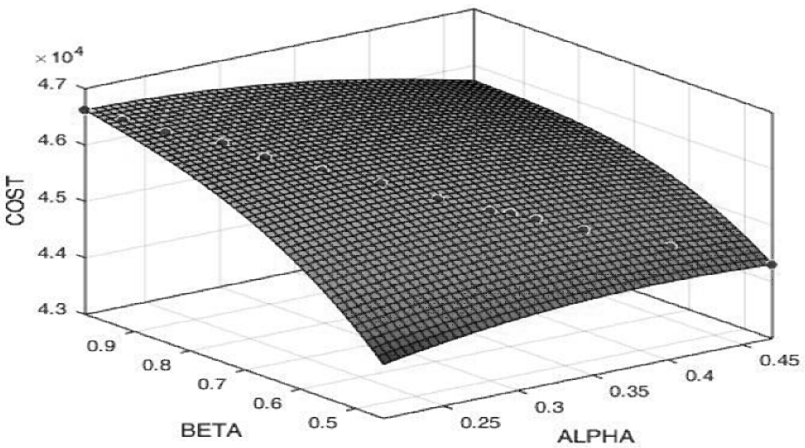


Fig. 11 expresses a three-dimensional view of average SC cost under the effect of primal and dual cuts. It is a surface like curve with concave nature. We see that the average SC cost becomes minimum ($ 43500) with respect to the value of primal-dual cut (α, β) = (0.26, 0.60) and it becomes maximum ($ 46500) with respect to the value of primal-dual cut (α, β) = (0.2, 0.95) respectively.

6. Conclusion

In this study we have discussed a noble application of fuzzy approximate reasoning in the field of inventory management problems. This article explores a new horizon on managerial decision making utilizing the approximating fuzzy mathematical modelling. To do this, the concept of possibility theory in approximating fuzzy membership function is incorporated. The concept of negation (non-membership degree) is applied within approximated fuzzy membership degree. Boosting the traditional approach of α-cuts of fuzzy sets, duality of α-cuts has been introduced over a rectangular hyperbolic type convex set. The basic managerial insights include as follows:

a) More choices have come to get decision at any time because of the presence of dual solution of the original objective function side by side.
b) Fuzzy reasoning solution approach gives nearly 8.5 % cost reduction on average with respect to crisp optimal solution.
c) Reducing length of cycle time, the DMs will have enough time to think further for any kind of strategic change.
d) Applying fuzzy reasoning it is possible to reduce individual differences of selecting particular fuzzy number and hence its global acceptance even less qualified DM also.

Conflict of interest

The authors declare that there is no conflict of interest regarding the publication of this article.

Acknowledgements

The authors are grateful to the anonymous reviewers, honorable Editor-in-Chief for their valuable suggestions and to improve the quality of the article.

References

1. Bhattacharya K, De SK, Khan A, Nayak PK, Pollution sensitive global crude steel production–transportation model under the effect of corruption perception index, OPSEARCH, 2021, 58, 636-660.
2. De SK, Solving an EOQ model under fuzzy Reasoning, Applied Soft Computing, 2020, https://doi.org/10.1016/j.asoc.2020.106892.
3. De SK, Triangular dense fuzzy lock sets, Soft Computing, 2017, https://doi.org/10.1007/s00500-017-2726-0.
4. De SK, Beg I, Triangular dense fuzzy sets and new defuzzification methods, Journal of Intelligent and Fuzzy Systems, 2016, 31(1), 469-477.
5. De SK, Mahata GC, Decision of a Fuzzy Inventory with Fuzzy Backorder Model under Cloudy Fuzzy Demand Rate, International Journal of Applied and Computational Mathematics, 2017, 3(3), 2593-2609.
6. De SK, Mahata GC, An EPQ model for three-layer supply chain with partial backordering and disruption: Triangular dense fuzzy lock set approach, Sadhana, 2019a, https://doi.org/10.1007/s12046-019-1160-7.
7. De SK, Mahata GC, A production inventory supply chain model with partial backordering and disruption under triangular linguistic dense fuzzy lock set approach, Soft Computing, 2019b, https://doi.org/10.1007/s00500-019-04254-2.
8. De SK, Mahata GC, A comprehensive study of an economic order quantity model under fuzzy monsoon demand, Sadhana, 2019c, https://doi.org/10.1007/s12046-019-1059-3.
9. De SK, Bhattacharya K , Roy B, Solution of a Pollution Sensitive Supply Chain Model under fuzzy approximate reasoning, Int.J. of Intelligent Systems, 2021a DOI: 10.1002/int.22522
10. De SK, Mahata GC , Maity S, Carbon emission sensitive deteriorating inventory model with trade credit under volumetric fuzzy system, Int.J. of Intelligent Systems, 2021b DOI: 10.1002/int.22599
11. De SK, Sana SS, Fuzzy order quantity inventory model with fuzzy shortage quantity and fuzzy promotional index, Economic Modelling, 2013a, 31, 351-358.
12. De SK, Sana SS, Backlogging EOQ model for promotional effort and selling price sensitive demand- an intuitionistic fuzzy approach, Annals of Operations Research, 2013b, https://doi.org/10.1007/s10479-013-1476-3.
13. De SK, Sana SS, Multi-criteria multi-attribute decision making for EOQ model under hesitant fuzzy environment, Pacific Science Review A: Natural Science and Engineering, 2015, 17(2), 61-68.
14. De SK, Sana SS, The (p,q,r,l) Model for

Stochastic Demand under Intuitionistic Fuzzy Aggregation with Bonferroni Mean, Journal of Intelligent Manufacturing, 2018, 29(8), 1753-1771.
15. De SK, Goswami A, Sana SS, An interpolating by pass to Pareto optimality in intuitionistic fuzzy technique for a EOQ model with time sensitive backlogging. Applied Mathematics and Computation, 2014, 230, 664-674.
16. Ebadi A, Moharram NN, Rahnamaee MT, Motesadi ZS, Determining the ecological footprint of vehicles in Tehran, Iran. Applied Ecology and Environmental Research, 2016, 14(3), 439-450.
17. Hernandez-Pellon A, Fernandez-Olmo I, Airborne concentration and deposition of trace metals and metalloids in an urban area downwind of a manganese alloy plant, Atmospheric Pollution Research, 2018, https://doi.org/10.1016/j.apr.2018.11.009.
18. Karmakar S, De SK, Goswami A, A pollution sensitive dense fuzzy economic production quantity model with cycle time dependent production rate, J. Clean. Prod., 2017, 154, 139-150.
19. Karmakar S, De SK, Goswami A, A pollution sensitive remanufacturing model with waste items: Triangular dense fuzzy lock set approach, J. Clean. Prod., 2018, 187, 789-803.
20. Mancini MS, Galli A, Niccolucci V, Lin D, Bastianoni S, Wackernagel M, Marchettini N, Ecological footprint: Refining the carbon footprint calculation, Ecological Indicators, 2016, 61, 390-403.
21. Nasseri SH, Ebrahimnejad A, A New Approach to Duality in Fuzzy Linear Programming, 2012, https://doi.org/10.1007/978-3-642-28592-9_2.
22. Ovchinnikov S, The duality principle in fuzzy set theory, Fuzzy sets and Systems, 1991, 42, 133-144.
23. Pal SK, Mandal DP, Fuzzy Logic and Approximate Reasoning: An Overview, IEEE Journal of Research, 1991, 37(5-6), 548-560.
24. Panigrahi M, Panda G, Nanda S, Convex fuzzy mapping with differentiability and its application in fuzzy optimization, European Journal of Operational Research, 2008, 185, 47–62.
25. Ramik J, Rimanek J, Inequality relation between fuzzy numbers and its use in fuzzy optimization, Fuzzy Sets and Systems, 1985, 16, 123-138.
26. Song CZ, Zhao YQ, Fuzzy multi-objective optimization of passive suspension parameters, Fuzzy Information and Engineering, 2010, 1, 87-100.
27. Tanaka H, Asai K, Fuzzy solutions in fuzzy linear programming problems, IEEE Transactions on Systems, Man and Cybernetics, 1984, SMC-14, 325-328.
28. Wu HC, Duality theory in fuzzy linear

programming problems with fuzzy coefficients. Fuzzy Optimization and Decision Making, 2003, 2 (1), 61-73.
29. Wu HC, Duality theorems and saddle point optimality conditions in fuzzy nonlinear programming problems based on different solution concepts, Fuzzy Sets and Systems, 2007, 158, 1588 – 1607.
30. Zadeh LA, Fuzzy sets, Information Control, 1965, 8(3), 338-356.
31. Zadeh LA, The concept of Linguistic variable and its application to approximate reasoning, Parts I, II, III, Information Sciences. 1975, 8, 199-251;8, 301-357; 9, 43-80.
32. Zhang C, Yung XH, Lee ES, Duality theory in fuzzy mathematical programming problems with fuzzy coefficients, Comput. Math. Appl., 2005, 49, 1709-1730.
33. Zimmermann HJ, Application of fuzzy set theory to Mathematical Programming, Information Sciences, 1985, 36, 29-58.
34. Zou T, Extended Duality in Fuzzy Optimization Problems, Mathematical Problems in Engineering, 2015, https://doi.org/10.1155/2015/826752.