<u>Mathematics Syllabus for M. Sc. Entrance Test</u>

Algebra: Polar representation of complex numbers, nth roots of unity, De Moivre's theorem for rational indices and its applications. Theory of equations: Relation between roots and coefficients, transformation of equation, Descartes rule of signs, cubic and biquadratic equation. Inequality: The inequality involving $AM \geq GM \geq HM$, Cauchy-Schwartz inequality. Equivalence relations. Functions, composition of functions, Invertible functions, one to one correspondence and cardinality of a set. Well-ordering property of positive integers, division algorithm, divisibility and Euclidean algorithm. Congruence relation between integers. Principles of Mathematical induction, statement of Fundamental Theorem of Arithmetic. Systems of linear equations, row reduction and echelon forms, vector equations, the matrix equation Ax=b, solution sets of linear systems, applications of linear systems, linear independence. Introduction to linear transformations, matrix of a linear transformation, inverse of a matrix, characterizations of invertible matrices. Subspaces ofRn , dimension of subspaces of Rn , rank of a matrix, Eigen values, eigen vectors and characteristic equation of a matrix. Cayley-Hamilton theorem and its use in finding the inverse of a matrix.

Differential Calculus: Hyperbolic functions, higher order derivatives, Leibnitz rule and its applications to problems of type eax+bsinx, eax+bcosx, (ax+b)n sinx, (ax+b)n cosx, concavity and inflection points, envelopes, asymptotes, curve tracing in cartesian coordinates, tracing in polar coordinates of standard curves, L'Hospital's rule, Reduction formulae, derivations and illustrations of reduction formulae of the type ∫sin nx dx, ∫cosnx dx, ∫tan nx dx, ∫sec nx dx, ∫(log x)n dx, ∫sinn x sinmx dx, parametric equations, parameterizing a curve, arc length of a curve, arc length of parametric curves, area under a curve, area and volume of surface of revolution, techniques of sketching conics.

Functions of several variables, limit and continuity of functions of two or more variables Partial differentiation, total differentiability and differentiability, sufficient condition for differentiability. Chain rule for one and two independent parameters, directional derivatives, the gradient, maximal and normal property of the gradient, tangent planes, Extrema of functions of two variables, method of Lagrange multipliers, constrained optimization problems. Double integration over rectangular region, double integration over non-rectangular region, Double integrals in polar co-ordinates, Triple integrals, triple integral over a parallelepiped and solid regions. Volume by triple integrals, cylindrical and spherical coordinates. Change of variables in double integrals and triple integrals. Definition of vector field, divergence and curl. Line integrals, applications of line integrals: mass and work. Fundamental theorem for line integrals, conservative vector fields, independence of path. Green's theorem, surface integrals, integrals over parametrically defined surfaces. Stoke's theorem, The Divergence theorem

Geometry: Reflection properties of conics, rotation of axes and second degree equations, classification of conics using the discriminant, polar equations of conics. Spheres. Cylindrical surfaces. Central conicoids, paraboloids, plane sections of conicoids, generating lines, classification of quadrics, illustrations of graphing standard quadric surfaces like cone, ellipsoid.

ODE and PDE: Differential equations and mathematical models. General, particular, explicit, implicit and singular solutions of a differential equation. Exact differential equations and integrating factors, separable equations and equations reducible to this form, linear equation and Bernoulli equations, special integrating factors and transformations. Lipschitz condition and Picard's Theorem (Statement only). General solution of homogeneous equation of second order, principle of super position for homogeneous equation, Wronskian: its properties and applications, Linear homogeneous and nonhomogeneous equations of higher order with constant coefficients, Euler's equation, method of undetermined coefficients, method of variation of parameters. Systems of linear differential equations, types of linear systems, differential operators, an operator method for linear systems with constant coefficients, Basic Theory of linear systems in normal form, homogeneous linear systems with constant coefficients: Two Equations in two unknown functions. Equilibrium points, Interpretation of the phase plane Power series solution of a differential equation about an ordinary point, solution about a regular singular point.

Partial differential equations – Basic concepts and definitions. Mathematical problems. First- order equations: classification, construction and geometrical interpretation. Method of characteristics for obtaining general solution of quasi linear equations. Canonical forms of first-order linear equations. Method of separation of variables for solving first order partial differential equations. Derivation of heat equation, wave equation and Laplace equation. Classification of second order linear equations as hyperbolic, parabolic or elliptic. Reduction of second order linear equations to canonical forms. The Cauchy problem, Cauchy-Kowalewskaya theorem, Cauchy problem of an infinite string. Initial boundary value problems. Semi-infinite string with a fixed end, semi-infinite string with a free end. Equations with non-homogeneous boundary conditions. Nonhomogeneous wave equation. Method of separation of variables, solving the vibrating string problem. Solving the heat conduction problem

Real Analysis: Review of algebraic and order properties of R, ε-neighborhood of a point in R. Idea of countable sets, uncountable sets and uncountability of R. Bounded above sets, bounded below sets, bounded sets, unbounded sets. Suprema and infima. Completeness property of R and its equivalent properties. The Archimedean property, density of rational (and Irrational) numbers in R, intervals. Limit points of a set, isolated points, open set, closed set, derived set,illustrations of Bolzano-Weierstrass theorem for sets, compact sets in R, Heine-Borel Theorem. Sequences, bounded sequence, convergent sequence, limit of a sequence, liminf, lim sup. Limit theorems. Monotone sequences, monotone convergence theorem. Subsequences, divergence criteria. Monotone subsequence theorem (statement only), Bolzano Weierstrass theorem for sequences. Cauchy sequence, Cauchy's convergence criterion. Infinite series, convergence and divergence of infinite series, Cauchy criterion, tests for convergence: comparison test, limit comparison test, ratio test, Cauchy's nth root test, integral test. Alternating series, Leibniz test. Absolute and conditional convergence.

Limits of functions (ε - δ approach), sequential criterion for limits, divergence criteria. Limit theorems, one sided limits. Infinite limits and limits at infinity. Continuous functions, sequential criterion for continuity and discontinuity. Algebra of continuous functions. Continuous functions on an interval, intermediate value theorem, location of roots theorem, preservation of intervals theorem. Uniform continuity, non-uniform continuity criteria, uniform continuity theorem. Differentiability of a function at a point and in an interval, Caratheodory's theorem, algebra of differentiable functions. Relative extrema, interior extremum theorem.

Rolle’s theorem. Mean value theorem, intermediate value property of derivatives, Darboux’s theorem. Applications of mean value theorem to inequalities and approximation of polynomials. Cauchy’s mean value theorem. Taylor’s theorem with Lagrange’s form of remainder, Taylor’s theorem with Cauchy’s form of remainder, application of Taylor’s theorem to convex functions, relative extrema. Taylor’s series and Maclaurin’s series expansions of exponential and trigonometric functions, ln (1 + x), 1/(ax + b) and(x+1)n . Application of Taylor’s theorem to inequalities.

Riemann integration: inequalities of upper and lower sums, Darbaux integration, Darbaux theorem, Riemann conditions of integrability, Riemann sum and definition of Riemann integral through Riemann sums, equivalence of two definitions. Riemann integrability of monotone and continuous functions, properties of the Riemann integral; definition and integrability of piecewise continuous and monotone functions. Intermediate Value theorem for Integrals; Fundamental theorem of Integral Calculus.

Improper integrals. Convergence of Beta and Gamma functions.

Pointwise and uniform convergence of sequence of functions. Theorems on continuity, derivability and integrability of the limit function of a sequence of functions. Series of functions; Theorems on the continuity and derivability of the sum function of a series of functions; Cauchy criterion for uniform convergence and Weierstrass M-Test. Fourier series: Definition of Fourier coefficients and series, Reimann Lebesgue lemma, Bessel's inequality, Parseval's identity, Dirichlet's condition. Examples of Fourier expansions and summation results for series. Power series, radius of convergence, Cauchy Hadamard theorem. Differentiation and integration of power series; Abel’s theorem; Weierstrass approximation theorem

Dynamics of Particle: Central force. Constrained motion, varying mass, tangent and normal components of acceleration, modelling ballistics and planetary motion, Kepler's second law.

Abstract Algebra: Symmetries of a square, dihedral groups, definition andexamples of groups including permutation groups and quaternion groups (through matrices), elementary properties of groups. Subgroups and examples of subgroups, centralizer, normalizer, center of a group, product of two subgroups. Properties of cyclic groups, classification of subgroups of cyclic groups. Cycle notation for permutations, properties of permutations, even and odd permutations, alternating group, properties of cosets, Lagrange’s theorem and consequences including Fermat’s Little theorem. External direct product of a finite number of groups, normal subgroups, factor groups, Cauchy’s theorem for finite abelian groups. Group homomorphisms, properties of homomorphisms, Cayley’s theorem, properties of isomorphisms. First, Second and Third isomorphism theorems. Automorphism, inner automorphism, automorphism groups, automorphism groups of finite and infinite cyclic groups, applications of factor groups to automorphism groups, Characteristic subgroups, Commutator subgroup and its properties. Properties of external direct products, the group of units modulo n as an external direct product, internal direct products, Fundamental theorem of finite abelian groups. Group actions, stabilizers and kernels, permutation representation associated with a given group action. Applications of group actions. Generalized Cayley’s theorem. Index theorem. Groups acting on themselves by conjugation, class equation and consequences, conjugacy in Sn, p-groups, Sylow’s theorems and consequences, Cauchy’s theorem, Simplicity of An for $n \geq 5$, non-simplicity tests.

Definition and examples of rings, properties of rings, subrings, integral domains and fields, characteristic of a ring. Ideal, ideal generated by a subset of a ring, factor rings, operations on ideals, prime and maximal ideals. Ring homomorphisms, properties of ring homomorphisms. Isomorphism theorems I, II and III, field of quotients. Polynomial rings over commutative rings, division algorithm and consequences, principal ideal domains, factorization of polynomials, reducibility tests, irreducibility tests, Eisenstein criterion, and unique factorization in Z [x]. Divisibility in integral domains, irreducible, primes, unique factorization domains, Euclidean domains.

Linear Algebra: Vector spaces, subspaces, algebra of subspaces, quotient spaces, linear combination of vectors, linear span, linear independence, basis and dimension, dimension of subspaces. Linear transformations, null space, range, rank and nullity of a linear transformation, matrix representation of a linear transformation, algebra of linear transformations. Isomorphisms. Isomorphism theorems, invertibility and isomorphisms, change of coordinate matrix. Dual spaces, dual basis, double dual, transpose of a linear transformation and its matrix in the dual basis, annihilators. Eigen spaces of a linear operator, diagonalizability, invariant subspaces and Cayley-Hamilton theorem, the minimal polynomial for a linear operator, canonical forms. Inner product spaces and norms, Gram-Schmidt orthogonalisation process, orthogonal complements, Bessel’s inequality, the adjoint of a linear operator. Least squares approximation, minimal solutions to systems of linear equations. Normal and self-adjoint operators. Orthogonal projections and Spectral theorem.

Numerical Analysis: Algorithms. Convergence. Errors: relative, absolute. Round off. Truncation. Transcendental and polynomial equations: Bisection method, Newton’s method, secant method, Regula-falsi method, fixed point iteration, Newton-Raphson method. Rate of convergence of these methods. System of linear algebraic equations: Gaussian elimination and Gauss Jordan methods. Gauss Jacobi method, Gauss Seidel method and their convergence analysis. LU decomposition. Interpolation: Lagrange and Newton’s methods. Error bounds. Finite difference operators. Gregory forward and backward difference interpolation. Numerical differentiation: Methods based on interpolations, methods based on finite differences. Numerical Integration: Newton Cotes formula, Trapezoidal rule, Simpson’s 1/3rd rule, Simpsons 3/8th rule, Weddle’s rule, Boole’s Rule. midpoint rule, Composite trapezoidal rule, composite Simpson’s 1/3rd rule, Gauss quadrature formula. The algebraic eigen value problem: Power method. Approximation: Least square polynomial approximation. Ordinary differential equations: The method of successive approximations, Euler’s method, the modified Euler method, Runge-Kutta methods of orders two and four.

Complex Analysis: Limits, limits involving the point at infinity, continuity. Properties of complex numbers, regions in the complex plane, functions of complex variable, mappings. Derivatives, differentiation formulas, Cauchy-Riemann equations, sufficient conditions for differentiability. Analytic functions, examples of analytic functions, exponential function, logarithmic function, trigonometric function, derivatives of functions, and definite integrals of functions. Contours, Contour integrals and its examples, upper bounds for moduli of contour integrals. Cauchy- Goursat theorem, Cauchy integral formula. Liouville’s theorem and the fundamental theorem of algebra. Convergence of sequences and series, Taylor series and its examples. Laurent series and its examples, absolute and uniform convergence of power series.