# Solutions of Vector Equations:

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Paper- C201

Course: Mathematics (H)

Prepared by: Dr. Ajoy Kumar Maiti

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## Solutions of Vector Equations :

**Ex-1** : Solve the equation $\vec{x}\cdot\vec{a} = p$ where $\vec{a}$ is a given vector and $p$ is a given scalar.

**Ans** : Given equation is $\vec{x}\cdot\vec{a} = p$.

$$\Rightarrow \vec{x}\cdot\vec{a} = p\,\frac{\vec{a}\cdot\vec{a}}{|\vec{a}|^2}$$

$$\Rightarrow \left(\vec{x} - \frac{p\vec{a}}{|\vec{a}|^2}\right)\cdot\vec{a} = 0 \quad (1)$$

We consider the general soln $\vec{x} = \frac{p\vec{a}}{|\vec{a}|^2} + \vec{h}\times\vec{a}$ which satisfy the equation (1), $\vec{h}$ is an arbitrary vector.

**Ex-2** : Solve the equation $p\vec{x} + (\vec{x}\cdot\vec{b})\,\vec{a} = \vec{c}$ $(p \neq 0)$ (1)

**Ans** : Multiplying the given equation on both sides scalarly by $\vec{b}$, we get

$$p\,\vec{x}\cdot\vec{b} + (\vec{x}\cdot\vec{b})(\vec{a}\cdot\vec{b}) = \vec{c}\cdot\vec{b} \quad (2).$$

$$\Rightarrow \vec{x}\cdot\vec{b} = \frac{\vec{c}\cdot\vec{b}}{p + \vec{a}\cdot\vec{b}} \quad [\because p + \vec{a}\cdot\vec{b} \neq 0]$$

Also from (1), $\vec{x}\cdot\vec{b} = \dfrac{\vec{c} - p\vec{x}}{\vec{a}}$ .... (3).

From (2) and (3) $\vec{c} - p\vec{x} = \dfrac{(\vec{c}\cdot\vec{b})\,\vec{a}}{p + \vec{a}\cdot\vec{b}}$

$$\Rightarrow \vec{x} = \frac{\vec{c}}{p} - \frac{(\vec{c}\cdot\vec{b})\,\vec{a}}{p(p + \vec{a}\cdot\vec{b})} \quad \text{provided } p + \vec{a}\cdot\vec{b} \neq 0$$

$$= \frac{\vec{c}}{p} - \lambda\vec{a}, \quad \lambda \text{ being parameter.}$$

This is the solution of the above equation.

**Ex-3** : Solve the equation $\vec{x}\times\vec{a} = \vec{b}$, $(\vec{a}\cdot\vec{b} = 0)$

**Ans** : Given equation is $\vec{x}\times\vec{a} = \vec{b}$.

Multiplying vectorially with $\vec{a}$ we have

$$\vec{a} \times (\vec{x} \times \vec{a}) = \vec{a} \times \vec{b}$$

$$\Rightarrow \vec{x}\,|\vec{a}|^2 - \vec{a}\,(\vec{a}\cdot\vec{x}) = \vec{a} \times \vec{b} \quad -(1)$$

The equation(1) is of the form $p\vec{x} + (\vec{x}\cdot\vec{b})\vec{a} = \vec{c}$

So, the soln of (1) is

$$x = \lambda\vec{a} + \frac{\vec{a}\times\vec{b}}{|\vec{a}|^2}, \quad \lambda \text{ being parameter.}$$

Otherwise : Suppose that the solution of the given equation is $\vec{x} = l\vec{a} + m\vec{b} + n(\vec{a}\times\vec{b})$ where $\vec{a}, \vec{b}, \vec{a}\times\vec{b}$ are non-coplanar vectors.

Given equation is $\vec{x} \times \vec{a} = \vec{b}$ (1)

Substituting the values of $\vec{x}$ in (1) we have

$$\{l\vec{a} + m\vec{b} + n(\vec{a}\times\vec{b})\} \times \vec{a} = \vec{b}$$

$$\Rightarrow m\,\vec{b}\times\vec{a} + n\,(\vec{a}\times\vec{b})\times\vec{a} = \vec{b}$$

$$\Rightarrow m\,\vec{b}\times\vec{a} + n\{(\vec{a}\cdot\vec{a})\vec{b} - (\vec{a}\cdot\vec{b})\vec{a}\} = \vec{b}$$

$$\Rightarrow -(1 - n|\vec{a}|^2)\vec{b} + m\,\vec{b}\times\vec{a} = 0 \quad [\because \vec{a}\cdot\vec{b} = 0]$$

$$\Rightarrow m = 0 \text{ and } 1 - n|\vec{a}|^2 = 0 \quad [\because \text{Since } \vec{b}, \vec{b}\times\vec{a} \text{ are non-coplanar vectors}]$$

$$\therefore n = \frac{1}{|\vec{a}|^2}.$$

$\therefore$ The general solution of (1) is $\vec{x} = l\vec{a} + \dfrac{(\vec{a}\times\vec{b})}{|\vec{a}|^2}$

$\boxed{\text{Ex-4}}$: Solve the vector equation $\vec{x}\times\vec{b} = \vec{a}\times\vec{b}$

Ans : Given equation is $\vec{x}\times\vec{b} = \vec{a}\times\vec{b}$

$$\Rightarrow (\vec{x} - \vec{a}) \times \vec{b} = 0$$

$\Rightarrow$ $\vec{x} - \vec{a}$ and $\vec{b}$ are parallel.

$\therefore$ $\vec{x} - \vec{a} = t\vec{b}$, $t$ being scalar.

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$\therefore \quad \vec{x} - \vec{a} = t\vec{b}$

$\Rightarrow \vec{x} = \vec{a} + t\vec{b}$.

This is the general solution of the above equation.

**Ex-5**: Solve the simultaneous equation $\vec{x} \times \vec{b} = \vec{c}$ and $\vec{x} \cdot \vec{a} = p$ $(\vec{a} \cdot \vec{b} \neq 0$

**Ans**: Given vector equations are $\vec{x} \times \vec{b} = \vec{c}$ and $\vec{x} \cdot \vec{a} = p$.

$$\vec{x} \times \vec{b} = \vec{c}$$

$$\Rightarrow \vec{a} \times (\vec{x} \times \vec{b}) = \vec{a} \times \vec{c}$$

$$\Rightarrow \vec{x}(\vec{a} \cdot \vec{b}) - \vec{b}(\vec{a} \cdot \vec{x}) = \vec{a} \times \vec{c}$$

$$\Rightarrow \vec{x}(\vec{a} \cdot \vec{b}) - \vec{b}\,p = \vec{a} \times \vec{c}$$

$$\left[\because \vec{x} \cdot \vec{a} = \vec{a} \cdot \vec{x} = p\right]$$

$$\Rightarrow x = \frac{\vec{b}p + \vec{a} \times \vec{c}}{\vec{a} \cdot \vec{b}} \quad \left[\because \vec{a} \cdot \vec{b} \neq 0\right]$$

This is the solution of the simultaneous equation.

**Ex-6**: Solve the simultaneous equation $\vec{x} \times \vec{b} = \vec{c} \times \vec{b}$, $\vec{x} \cdot \vec{a} = 0$.
$(\vec{a} \cdot \vec{b} \neq 0)$.

**Ans**: Given equation is $\vec{x} \times \vec{b} = \vec{c} \times \vec{b}$

$$\Rightarrow (\vec{x} - \vec{c}) \times \vec{b} = 0$$

$$\Rightarrow \vec{x} - \vec{c} = t\vec{b} \quad \left[\because \vec{x} - \vec{c} \text{ is } \parallel \text{ to } \vec{b}\right]$$

$t$ being scalar.

$$\Rightarrow \vec{x} = \vec{c} + t\vec{b}. \quad \text{—(1)}$$

Substituting this value in $\vec{x} \cdot \vec{a} = 0$

$$\Rightarrow (\vec{c} + t\vec{b}) \cdot \vec{a} = 0$$

$$\Rightarrow t = -\frac{\vec{c} \cdot \vec{a}}{\vec{a} \cdot \vec{b}} \quad (\because \vec{a} \cdot \vec{b} \neq 0)$$

Hence we have $\vec{x} = \vec{c} - \dfrac{\vec{c} \cdot \vec{a}}{\vec{a} \cdot \vec{b}} \vec{b}$.

**Theorem:** The necessary and sufficient condition that the vector equation $\vec{a}\times\vec{x}=\vec{b}$, where $\vec{a}$ and $\vec{b}$ are given vectors and $\vec{a}\neq 0$ possesses a solution is that $\vec{a}\cdot\vec{b}=0$.

**Proof:** The condition is necessary:

We have $\vec{a}\cdot\vec{b}=\vec{a}\cdot(\vec{a}\times\vec{x})=0$ $\quad [\vec{b}=\vec{a}\times\vec{x}]$

So, the condition is necessary.

The condition is sufficient:

Let $\vec{a},\vec{b}$ and $\vec{b}\times\vec{a}$ are non-coplanar vectors so, $\vec{x}$ can be expressed as $\vec{x}=l\vec{a}+m\vec{b}+n\,\vec{b}\times\vec{a}$.

We have $\vec{a}\times\vec{x}=\vec{b}$.

$$\Rightarrow \vec{a}\times(l\vec{a}+m\vec{b}+n\,\vec{b}\times\vec{a})=\vec{b}$$

$$\Rightarrow m\,\vec{a}\times\vec{b}+n\{\vec{a}\times(\vec{b}\times\vec{a})\}=\vec{b}$$

$$\Rightarrow m\,\vec{a}\times\vec{b}+n\{\vec{b}(\vec{a}\cdot\vec{a})-\vec{a}(\vec{a}\cdot\vec{b})\}=\vec{b}.$$

$$\Rightarrow m\,\vec{a}\times\vec{b}+n\,\vec{b}\,|\vec{a}|^2=\vec{b} \quad [\vec{a}\cdot\vec{b}=0]$$

$$\Rightarrow m\,\vec{a}\times\vec{b}+(n|\vec{a}|^2-1)\,\vec{b}=0.$$

Since $\vec{a}\times\vec{b}$ and $\vec{b}$ are non-coplanar.

$$\therefore\ m=0 \text{ and } n|\vec{a}|^2-1=0$$

$$\Rightarrow n=\frac{1}{|\vec{a}|^2}.$$

$$\therefore\ \vec{x}=l\vec{a}+\frac{\vec{b}\times\vec{a}}{|\vec{a}|^2},$$ $l$ being the scalar.

This is the general solution of $\vec{a}\times\vec{x}=\vec{b}$.

Hence the theorem.

$\boxed{\text{Ex-7}}$: Find the scalars $l$ and $m$ such that $l\vec{a}+m\vec{b}=\vec{c}$, $\vec{a},\vec{b},\vec{c}$ being given vectors.

Given equation is $l\vec{a} + m\vec{b} = \vec{c}$ . (1)

Multiplying (1) vectorially by $\vec{b}$ ~~unto~~ we have

$$(l\vec{a} + m\vec{b}) \times \vec{b} = \vec{c} \times \vec{b}$$

$$\Rightarrow l\,\vec{a}\times\vec{b} = \vec{c}\times\vec{b}$$

$$\Rightarrow l\,|\vec{a}\times\vec{b}|^2 = (\vec{c}\times\vec{b})\cdot(\vec{a}\times\vec{b}).$$

$$\Rightarrow l = \frac{(\vec{c}\times\vec{b})\cdot(\vec{a}\times\vec{b})}{|\vec{a}\times\vec{b}|^2}$$

Also, multiplying (1) vectorially by $\vec{a}$ we have

$$\cancel{m\vec{b}\times\vec{a}} \quad (l\vec{a} + m\vec{b}) \times \vec{a} = \vec{c}\times\vec{a}$$

$$\Rightarrow m\,\vec{b}\times\vec{a} = \vec{c}\times\vec{a}$$

$$\Rightarrow m\,|\vec{b}\times\vec{a}|^2 = (\vec{c}\times\vec{a})\cdot(\vec{b}\times\vec{a}).$$

$$\Rightarrow m = \frac{(\vec{c}\times\vec{a})\cdot(\vec{b}\times\vec{a})}{|\vec{b}\times\vec{a}|^2}.$$

$\boxed{\text{Ex-8}}$ : Solve $\vec{a}\times\vec{x} = \vec{c}$ and $\vec{a}\cdot\vec{x} = k$ where $\vec{a}$ and $\vec{c}$ are any two vectors and $k$ is a scalar.

Ans : Given that $\vec{a}\times\vec{x} = \vec{c}$ —(1)

and $\vec{a}\cdot\vec{x} = k$ —(2)

Multiplying ~~(1)~~ scalarly (1) by $\vec{a}$ we have

$$\vec{a}\cdot(\vec{a}\times\vec{x}) = \vec{a}\cdot\vec{c}$$

$$\Rightarrow \vec{a}\cdot\vec{c} = 0 . \quad \text{—(3)}.$$

Also, $\vec{a}\times\vec{x} = \vec{c}$

$$\Rightarrow (\vec{a}\times\vec{x})\times\vec{a} = \vec{c}\times\vec{a}$$

$$\Rightarrow |\vec{a}|^2\vec{x} - (\vec{a}\cdot\vec{x})\,\vec{a} = \vec{c}\times\vec{a}$$

$$\Rightarrow \vec{x} = \frac{\vec{c}\times\vec{a} + k\vec{a}}{|\vec{a}|^2} \quad [\because \vec{a}\cdot\vec{x} = k] \quad [\because \vec{a}\neq 0]$$

$\therefore$ Therefore, the above simultaneous equation has a solution if $\vec{a}\cdot\vec{c} = 0$.

Rem: If $\vec{a}\cdot\vec{c}\neq 0$, there is no solution, since $\vec{a}\cdot(\vec{a}\times\vec{x}) = 0$. Hence the equation $\vec{a}\times\vec{x} = \vec{c}$ is inconsistent.

If $\vec{a} = 0$, there is no solution unless $\vec{c} = 0$ and $k = 0$. In this case $\vec{v}$ can be any vector.

**Ex-9**: Find the vector $\vec{x}$ from the equation $\vec{x}\times\vec{\beta} = \vec{r}$ and $\vec{x}\cdot\vec{\alpha} = 3$ where $\vec{\alpha} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{\beta} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{r} = -4(\hat{j} + \hat{k})$.

Ans: Given equation is $\vec{x}\times\vec{\beta} = \vec{r}$

$$\Rightarrow \vec{\alpha}\times(\vec{x}\times\vec{\beta}) = \vec{\alpha}\times\vec{r}$$

$$\Rightarrow \vec{x}\,(\vec{\alpha}\cdot\vec{\beta}) - \vec{\beta}\,(\vec{\alpha}\cdot\vec{x}) = \vec{\alpha}\times\vec{r} \quad \text{---(1)}$$

Now $\vec{\alpha}\times\vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & -4 & -4 \end{vmatrix} = \hat{i}(-8+4) + 4\hat{j} - 4\hat{k}$.

$\vec{\alpha}\cdot\vec{\beta} = 1$.

$\therefore$ From (1) $\vec{x} - 3\beta = -4\hat{i} + 4\hat{j} - 4\hat{k}$ $[\because \vec{x}\cdot\vec{\alpha} = 3]$

$$\Rightarrow \vec{x} = 3(2\hat{i} - \hat{j} + \hat{k}) - 4\hat{i} + 4\hat{j} - 4\hat{k}$$
$$= 2\hat{i} + \hat{j} - \hat{k}.$$

$\therefore$ $\vec{x} = 2\hat{i} + \hat{j} - \hat{k}$ is the solution of the above equation.

**Ex-10**: Show that the solution of the equation $k\vec{r} + \vec{r}\times\vec{a} = \vec{b}$ where $K$ is a non zero scalar and $\vec{a}$ and $\vec{b}$ are two vectors, can be put as $\vec{r} = \frac{1}{k^2 + |\vec{a}|^2}\left[\frac{\vec{a}\cdot\vec{b}}{k}\,\vec{a} + k\vec{b} + \vec{a}\times\vec{b}\right]$.

Ans: Suppose $\vec{a}, \vec{b}, \vec{a}\times\vec{b}$ are non-co-planar vectors. So, $\vec{r}$ can be expressed as a linear combination of these three non-coplanar vectors.

$\therefore \vec{r} = x\vec{a} + y\vec{b} + z\,\vec{a}\times\vec{b}$, $x, y, z$ being scalar.

Given that $k\vec{r} + \vec{r}\times\vec{a} = \vec{b}$

$$\Rightarrow k\left[x\vec{a} + y\vec{b} + z\,\vec{a}\times\vec{b}\right] + (x\vec{a} + y\vec{b} + z\,\vec{a}\times\vec{b})\times\vec{a} = \vec{b}$$

$$\Rightarrow kx\vec{a} + ky\vec{b} + kz\,\vec{a}\times\vec{b} + y\vec{b}\times\vec{a} + z\{(\vec{a}\times\vec{b})\times\vec{a}\} = \vec{b}$$

$$\Rightarrow kx\vec{a} + ky\vec{b} + kz\,\vec{a}\times\vec{b} - y\,\vec{a}\times\vec{b} + z\{\vec{b}|\vec{a}|^2 - \vec{a}(\vec{a}\cdot\vec{b})\} = \vec{b}$$

$$\Rightarrow \vec{a}\,(kx - z\,\vec{a}\cdot\vec{b}) + \vec{b}\,(ky + z|\vec{a}|^2 - 1) + \vec{a}\times\vec{b}\,(kz - y) = 0$$

Since $\vec{a}, \vec{b}, \vec{a}\times\vec{b}$ are non-coplanar vectors,

$\therefore kx - z\,\vec{a}\cdot\vec{b} = 0$, —(1) $\quad ky + z|\vec{a}|^2 - 1 = 0$ —(2)

and $kz - y = 0$ —(3).

From (3) $y = kz$.

From (2), $k^2 z + z|\vec{a}|^2 - 1 = 0 \Rightarrow z = \dfrac{1}{k^2 + |\vec{a}|^2}$

$$\therefore y = \frac{k}{k^2 + |\vec{a}|^2}.$$

From (1), $x = \dfrac{z\,\vec{a}\cdot\vec{b}}{k}$

$$= \frac{\vec{a}\cdot\vec{b}}{k(k^2 + |\vec{a}|^2)}.$$

$$\therefore \vec{r} = \frac{\vec{a}\cdot\vec{b}}{k(k^2+|\vec{a}|^2)}\vec{a} + \frac{k}{k^2+|\vec{a}|^2}\vec{b} + \frac{\vec{a}\times\vec{b}}{k^2+|\vec{a}|^2}$$

$$= \frac{1}{k^2+|\vec{a}|^2}\left[\frac{\vec{a}\cdot\vec{b}}{k}\vec{a} + k\vec{b} + \vec{a}\times\vec{b}\right] \text{ (Proved)}$$

**Ex-11**: If $\vec{a}\times\vec{r} = \vec{b}+\lambda\vec{a}$ and $\vec{a}.\vec{r} = 3$ where $\vec{a} = 2\hat{i}+\hat{j}-\hat{k}$ and $\vec{b} = -\hat{i}-2\hat{j}+2\hat{k}$, then show that $\vec{r} = \hat{i}+\hat{j}$. and $\lambda = 1$.

Ans: Given that $\vec{a}\times(\vec{a}\times\vec{r}) = \vec{a}\times\vec{b} + \lambda\vec{a}\times\vec{a}$ $\lambda$ being scalar. [Multiplying vectorially by $\vec{a}$ on both sides].

$\Rightarrow \vec{a}(\vec{a}.\vec{r}) - \vec{r}(\vec{a}.\vec{a}) = \vec{a}\times\vec{b}$.

$\Rightarrow 3\vec{a} - 6\vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ -1 & -2 & 2 \end{vmatrix}$. $\left[\because \vec{a}.\vec{r} = 3,\ |\vec{a}| = \sqrt{6}\right]$

$\Rightarrow 6\vec{r} = 3(2\hat{i}+\hat{j}-\hat{k}) + 3\hat{j} + 3\hat{k}$ $\left[\because \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ -1 & -2 & 2 \end{vmatrix} = -3\hat{j}-3\hat{k}\right]$

$\Rightarrow \vec{r} = \hat{i}+\hat{j}$.

2nd Part: $\vec{a}\times\vec{r} = \vec{b}+\lambda\vec{a}$.

Multiplying scalarly by $\vec{a}$ we have

$$\vec{a}.(\vec{a}\times\vec{r}) = \vec{a}.\vec{b} + \lambda\,\vec{a}.\vec{a}$$

$$\Rightarrow 0 = \vec{a}.\vec{b} + \lambda|\vec{a}|^2$$

$$\Rightarrow -6\lambda = -6$$

$$\Rightarrow \lambda = 1.$$

$\therefore$ The solution is $\vec{r} = \hat{i}+\hat{j}$ and $\lambda = 1$.

**Ex-12**: If $\vec{r}\times\vec{b} = \vec{c}\times\vec{b}$ and $\vec{r}.\vec{a} = 0$ where $\vec{a} = 2\hat{i}+3\hat{j}-\hat{k}$, $\vec{b} = 3\hat{i}-\hat{j}+\hat{k}$, $\vec{c} = \hat{i}+\hat{j}+3\hat{k}$. then show that $\vec{r} = 2(-\hat{i}+\hat{j}+\hat{k})$.

Ans: Given that $\vec{r}\times\vec{b} = \vec{c}\times\vec{b}$

$$\Rightarrow (\vec{r}-\vec{c})\times\vec{b} = 0$$

$\Rightarrow \vec{r} = c + t\vec{b}$ [$\because \vec{r}-\vec{c}$ and $\vec{b}$ are parallel]

Multiplying scalarly by $\vec{a}$ we have

$$\vec{r}.\vec{a} = \vec{c}.\vec{a} + t\,\vec{b}.\vec{a}$$

$\Rightarrow 0 = 2+2t$ [$\because \vec{r}.\vec{a} = 0,\ \vec{c}.\vec{a} = 2,\ \vec{b}.\vec{a} = 2$]

$\Rightarrow t = -1.$

$\therefore \vec{p} = \vec{c} - \vec{b}$

$= (\hat{i} + \hat{j} + 3\hat{k}) - (3\hat{i} - \hat{j} + \hat{k})$

$= -2\hat{i} + 2\hat{j} + 2\hat{k}$

$\therefore \vec{p} = 2(-\hat{i} + \hat{j} + \hat{k})$. (Proved).

$\boxed{\text{Ex-12}}$: Show that for the equations, $\vec{x}\cdot\vec{a} = l$, $\vec{x}\cdot\vec{b} = m$, $\vec{x}\cdot\vec{c} = n$

$x = l\vec{a}' + m\vec{b}' + n\vec{c}'$ where $(\vec{a}, \vec{b}, \vec{c})$ and $(\vec{a}', \vec{b}', \vec{c}')$ are reciprocal.

Ans: Let $\vec{a}, \vec{b}, \vec{c}$ be three non-coplanar vectors. Then their reciprocal vectors $\vec{a}', \vec{b}', \vec{c}'$ are also non-co-planar vectors.

So any vector can be expressed as the linear combination of $\vec{a}', \vec{b}', \vec{c}'$.

Hence, $\vec{x} = l\vec{a}' + m\vec{b}' + n\vec{c}'$ —(1)

As the vectors $(\vec{a}, \vec{b}, \vec{c})$ and $(\vec{a}', \vec{b}', \vec{c}')$ are reciprocal.

$$\therefore \left.\begin{array}{l} \vec{a}\cdot\vec{a}' = \vec{b}\cdot\vec{b}' = \vec{c}\cdot\vec{c}' = 1 \\ \text{and } \vec{a}'\cdot b = \vec{a}'\cdot c = 0 = \vec{a}\cdot\vec{b}' = \vec{a}\cdot\vec{c}'.\ \text{etc} \end{array}\right\} (2)$$

Now, $(l\vec{a}' + m\vec{b}' + n\vec{c}')\cdot\vec{a}$

$= l\,\vec{a}'\cdot\vec{a} + m\,\vec{b}'\cdot\vec{a} + n\,\vec{c}'\cdot\vec{a}$

$= l$ [using (2)].

$\therefore \vec{x} = l\vec{a}' + m\vec{b}' + n\vec{c}'$ satisfy the equation $\vec{x}\cdot\vec{a} = l$.

Similarly, $\vec{x} = l\vec{a}' + m\vec{b}' + n\vec{c}'$ satisfies $\vec{x}\cdot\vec{b} = m$, $\vec{x}\cdot\vec{c} = n$.

So $\vec{x} = l\vec{a}' + m\vec{b}' + n\vec{c}'$ is the solution of the equations

$\vec{x}\cdot\vec{a} = l$, $\vec{x}\cdot\vec{b} = m$ and $\vec{x}\cdot\vec{c} = n$

where $(\vec{a}, \vec{b}, \vec{c})$ and $(\vec{a}', \vec{b}', \vec{c}')$ are reciprocal to each other.

Ex-15 : Show that the vector equation $\vec{x} \times \vec{a} + (\vec{x} \cdot \vec{b})\vec{c} = \vec{d}$ is satisfied if $\vec{x} = \lambda \vec{a} + \vec{a} \times \dfrac{\vec{a} \times (\vec{d} \times \vec{c})}{(\vec{a} \cdot \vec{c})\,|\vec{a}|^2}$.

Ans : Try yourself :